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FEP FACULDADE DE ECONOMIA
UNIVERSIDADE DO PORTO

FDI under political risk and competition, a real options approach

Bruno Duarte Durães Pinto

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Paulo Pereira, PhD

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Abstract

We develop a model to understand the impact of political risk on the investment timing and size for firms. Considering political risk as being synonymous with expropriation, we find that in a market where only one firm operates (monopoly), expropriation risk does not affect the investment size, only affects the timing. For the case of the monopolist it will make him invest sooner than normal.

When we look at a competitive setting we see that, making the assumption that firms are symmetric and only invest if the government is not operating in the market, the leader will tend to invest later with more quantity and the follower will invest sooner with a lower commitment.

We also find that the government, in practice, sets a price ceiling that once hit, makes it expropriate the firms in the market.

When we allow for the possibility for producing in the home country, we see that firms will tend to produce in their home country if the amount of compensation is bellow a compensation threshold for expropriation. If firms choose to produce in their home country they will delay further the investment when compared to the case for producing in the foreign country, and the market price will be higher comparatively.

Key Words: Real-Options, Option-Games, FDI, Expropriation

JEL-Codes: C73, D81, F21, G31, L13

Sumário

Desenvolvemos um modelo para entender o impacto do risco político na temporização e dimensão do investimento das empresas. Considerando risco político como sendo sinônimo de expropriação, verificamos que num mercado onde uma única firma opera (monopólio), o risco de expropriação não afeta o tamanho do investimento, somente afeta a temporização. No caso do monopolista verificamos que o fará investir mais cedo do que o normal.

Quando olhamos para um cenário competitivo, assumindo que as empresas são simétricas e somente investem se o governo não estiver ativo no mercado, o líder tende a investir mais tarde e em maior quantidade e o seguidor irá investir mais cedo com menor compromisso.

Também descobrimos que o governo, na prática, estabelece um teto de preços que, uma vez atingido, o faz expropriar as empresas no mercado.

Quando permitimos a possibilidade de produzir no país de origem, vemos que as empresas tenderão a produzir no seu país de origem se o valor da compensação estiver abaixo de um valor *threshold*. Se as empresas optarem por produzir no seu país de origem, adiaram ainda mais o investimento em comparação com o caso de produção no país estrangeiro, e o preço de mercado será mais alto comparativamente.

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Chapter 1

Introduction

When we turn on the TV, we seem to always ear a government representative talk about attracting investment. The idea of having multinational enterprises (MNE's) investing in one's country can be synonymous with growth, technological advances, and knowledge spillover.

But when should firms invest in foreign countries? When is it optimal for MNE to invest?

When MNEs decide to invest abroad, known as Foreign Direct Investment (FDI), generally is so the firm can enter a new market (foreign-market-seeking FDI), to shift production to a place that allows for lower overall costs (efficiency-seeking FDI) or to a location where the necessary resources are available (resource-seeking FDI) (Buckley et al., 2007).

One thing that all of these types of investments have in common is the uncertainty concerning the outcomes. For example, a firm investing abroad with the goal of obtaining a piece of the new market does not know exactly the evolution of the market that will take place once the investment is made. The cost of service, the demand and the growth of the host country market are some of the very important considerations that MNEs have to take when doing FDI. (Buckley and Casson, 1981).

When planning to enter a new market, a firm also needs to take into account potential competition. If a market is still to be explored, having an expectation when competition will enter and the monopolistic rents will end, and conversely when to enter a market when there is an incumbent firm, can alter the amount that a firm is willing to supply to the market. Firms need to "account what they think will be the other firms' reactions to their own investment actions and realize that their competitors think the same way" (Azevedo and Paxson, 2014). A market might have growth potential but if it is already heavily populated no economical profits will be found.

Another factor that MNEs take into account when investing is the political risk in the host country (Jeanneret, 2016). A firm investing abroad in a country where the local government has the propensity for investment expropriation, or even worse for confiscation, may change the way investments are made. Such events can be seen recently in Latin America where populist governments started a wave of nationalizations.

Political transparency and high reputational costs suffered from expropriation by governments are an indirect insurance for a smooth investment. Of course not every host country can be considered political safe or even transparent.

Gastanaga, Nugent, and Pashamova (1998) in their study of policies/institutional variables that affect FDI found that, with a sample of 22 Least Developed Countries (LDC's), nationalization risk was significant in deterring FDI. These results are comprehensive since it is expected that no company wants its investments nationalized. Expropriation risk generally depends on the time horizon of political regimes, as a sufficiently small time horizon will make the expropriation of an asset more appreciable than the yield from taxes from that asset, and so governments will be more willing to violate property and contract rights (Clague, Keefer, Knack, and Olson, 1996).

Market entry also plays a role in a firm's decision. The "how to do it" question either throw exporting, wholly owned subsidiary, licensing or joint venture does play a role when specific costs and technological requirements may favor some over others (Buckley and Casson, 1998a).

The aim of this study is to primarily combine the market uncertainty and political risk in the FDI context and give insights into how firms should go about when faced with these unknowns.

This study, using the Real Option (RO) and Real Options Games theory (ROG) will try to answer a simple question: In the FDI context, when is the optimal time to invest when there exists competition and political risk?

Chapter 2 of this dissertation contains a literature review of the subject; in Chapter 3 the base case is laid out and in Chapter 4 is extended to include the risk of expropriation; Chapter 5 includes the possibility of producing in the home country to then export to the foreign market and Chapter 6 gives the final conclusions.

Chapter 2

Literature Review

Since the seminal work of Black and Scholes (1973) and Merton (1973) on the theory of option pricing, the literature on Real Options, a term introduced by Myers (1977) to distinguish the opportunities (or options) to buy real assets, has been growing at a rapid pace and aims to fill a gap that general theories do not account for, the flexibility embedded in the decision making process. Myers (1977) was a pioneer in transitioning the theory of options pricing to the corporate finance world. By using the real options approach it is possible to value the flexibility that otherwise would not be accounted for in the classic framework of capital budgeting (the Net Present Value (NPV) approach is the case in point). As noted by McDonald and Siegel (1986), investment timing is very important and not accounting for the existing flexibility in a given project and just rely on traditional methods (NPV) can produce suboptimal investments. Dixit and Pindyck (1994) and subsequently Trigeorgis (1996) provide useful and insightful textbooks for the theory and applications of real options.

As Buckley and Casson (1998b) indicate, the static approaches still have a role in helping to build more dynamic models that account for market uncertainty and flexibility. New methodologies to model FDI nowadays have to take into consideration a world that shifts and moves and bring static approaches to a more realistic dynamic one.

2.1 Real Option's Theory applied to FDI

Generally speaking, RO literature on FDI tends to focus on the optimal choice of entry. For example, Capel (1992) focuses on the uncertainty of the market-servicing costs and the possible adjustment costs when switching between servicing modes. He found that uncertainty postpones switching even at certain levels where cost savings are higher than switching costs. Furthermore, he also found that when uncertainty is high it is optimal to choose a flexible market entry mode characterized by higher production cost but lower adjustment costs because this mode can lead to lower total costs, eventhou it carries higher production costs. On the same line of thought Buckley and Casson (1998a) also found that lower level of commitment is preferred when uncertainty is high.

The entry mode under uncertain profits, different tax rates between home and the host country, cost advantages, institutional requirements and degree of cooperation in joint ventures will give different answers to what is the best way to enter a market (Pennings and Sleuwaegen, 2004).

Li and Rugman (2007) find that MNE's are more inclined to invest in their home country if they perceive that investing abroad does not generate additional real options or when the real options in the foreign country are difficult to exercise. Li and Rugman (2007) also find that when uncertainty is high, firms tend to make low commitment investments (e.g. licensing) which is in line with previous studies.

2.2 Modeling competition with Option Games

Game theory can be traced back to 1838 in the well-known industrial organization model of Cournot competition and alike. But the big stride has made by John Nash in the 1950's with the introduction of the more general theory of non-cooperative games.

In the RO framework, early literature considered competition as being an exogenous event (e.g. Baldwin, 1982).

ROG literature at the moment contains games that can be played as "one-shot", zero-sum, simultaneous/sequential or cooperative/non-cooperative and with perfect/imperfect, complete/incomplete, symmetric/asymmetric information. ROG extends the standard game theory by making the player's payoffs evolve stochastically instead of being static.

The present work is more interested in sequential, non-cooperative, with perfect and symmetric information in a zero-some preemption game, for simplicity sake.

From the general RO theory we know that when a firm has a monopoly over an investment decision, the firm also has an option to delay that invest and wait for new information. In competition, where there are multiple firms with the same investment opportunity, such value in waiting for new information is eroded by the prospect of pre-emption (Fudenberg and Tirole, 1985).

It was shown by Reinganum (1981) that in a duopoly with identical firms and perfect information, with the firms having a shared option, a Nash-equilibrium exists where a sequential investment type game will occur rather than simultaneously. In his paper, however, firms act "naïvely", not accounting for other firm's actions, reaching an open-loop equilibrium.

In contrast, Fudenberg and Tirole (1985) considering closed-loop strategies that allow for the consideration of previous actions, showed that when there is a first mover advantage, rents (payoffs) are equalized, meaning that because of threat of being preempted, and thus losing the first mover advantage, the decision to invest will come at the first point where the option for leader and follower have the same value and thus both are indifferent. In this setting, a firm will invest first preempting the other, and the second one will invest when its "follower's option" to invest is maximized. Once the second firm invests the first mover advantage is diluted and both share the market.

Grenadier (2002) derived a Cournot-Nash equilibrium model with a constant elasticity inverse demand function with a multiplicative demand shock following a

Geometric Brownian Motion (GBM) with symmetric firms producing one homogeneous good in an oligopolistic setting. He showed that competition drastically erodes the cushion that a monopolistic firm can create before committing and that investment in a competitive setting can lead to close to zero NPV investments.

Although the "cushion" may be reduced by the existence of competition, higher levels of uncertainty can lead the entrant, second investor, to delay its investment as it waits for more information, leaving a longer monopoly period for the leader (Huisman and Kort, 2015). If, in a duopolistic setting, and products offered by the symmetric firms are substitutes, the leader overinvests, coupled with uncertainty, it will have the effect of delaying the entrant's investment decision as it will invest less and later, than if the leader didn't overinvest, lengthening the monopolistic rents for the leader (Huisman and Kort, 2015).

The first known literature combining RO with game theory was developed by Smets (1993) in the context of FDI, extending the framework of Fudenberg and Tirole (1985) by adding uncertainty in a duopoly setting where the two ex-ante symmetric firms have a shared option to expand.

ROG's can be seen throw R&D, patents, natural resources, technology adoption and other areas in the literature [1], but, to the best of my knowledge, there seems to not exist any other published literature regarding FDI that explores a competitive setting with political risk included.

Chevalier-Roignant and Trigeorgis (2011) book provides a well-structured literature review with extensions of the literature regarding ROG, but, as is the general case for RO literature, the investment size is fixed, not accounting for the optimal magnitude of investment for the given level of demand. Huisman and Kort (2015) account for this issue, using a model that accounts for the optimal investment size.

2.3 Political risk

In this setup, it is considered that political risk is synonymous with expropriation risk. As pointed out by Cole and English (1991), governments may expropriate either for opportunistic or desperation reasons. Opportunistic reasons come when consumption gains from expropriation make the utility gains from expropriation the largest, or in other words governments expropriate given the high return on capital. In turn, desperation actions come when the largest utility gain occurs when consumption is low and the country marginal utility from expropriation is high. A welfare maximizer government chooses to expropriate if the act of expropriation increases current welfare.

MNE's doing FDI are more exposed than firms only doing domestic investments to foreign exchange risk and country risk (e.g. environmental, political) (Mahajan, 1990). Mahajan (1990), using the contingent claims analysis, prices expropriation risk as an American non-dividend paying call option that an MNE writes to the host country when it undertakes FDI. When the host country exercises such option it pays the exercise price (i.e. compensation).

Using a different approach, Nordal (2001) used country risk indices as a stochastic

[1] For a review of literature see Azevedo and Paxson (2014)

variable for a more general measure of country risk (e.g. economic, commercial, political) making the connection between risk indices and the underlying investment (oil in the case). The model is whoever dependent on the level of correlation between, in the case, oil and the risk indices, and has pointed out by the author few estimated correlation coefficients were found to be statically significant.

Clark (2003), extending the work of Mahajan (1990), introduced dividends, continuous exercise and models the firm's position explicitly and, similarly to Clark (1997), the cost of expropriation risk is treated as the value of an insurance policy that pays off all losses resulting from expropriation. In it, the firm expropriation risk is seen as a government trying to optimize its expropriation option. Similarly, Schwartz and Trolle (2010) study expropriation in natural resources projects as a government with the option to expropriate an oil field and thus receives all futures profits losing the foregone taxes and/or royalties. In their paper, they extend the literature including other costs of expropriating, namely inefficient public management and reputational costs. They postulate that the value of expropriation rises with the spot price, futures slope curve and volatility and decreases the higher the corporate tax and expropriation costs. Stroebel and van Benthem (2013) also postulate that higher taxes lower the risk of expropriation.

Restrepo Ochoa, Correia, Peña, and Población (2015) extended the literature by incorporating the effect of expropriation risk in the decision to abandon the investment and delved deeper in the expropriation cost for the government and for the overall economy with the given impact in the reputation. They formulate indemnity and reputational costs as an endogenous problem (in Clark (2003) and Schwartz and Trolle (2010) this problem is exogenous) by linking the reputational costs to compensation for expropriation for a welfare maximizing government and postulate that the welfare-maximizing government should "always offer the highest possible compensation to the target firm to minimize welfare losses". Restrepo Ochoa et al. (2015) will serve as a guideline for the political risk problem in this work.

Chapter 3

Safe Environment

The main goal in this Chapter is to create a base case, without risk of expropriation, that develops the basic ideas to then be extended in the following Chapter to accommodate the risk of expropriation. The basic setup is a blend of the market and optimal investment decision portion in Huisman and Kort (2015) with the firms cost structure and tax scheme in Restrepo Ochoa et al. (2015). The setup and ideas described in this Chapter will be transfer to the next Chapter to be built upon.

Similar to Huisman and Kort (2015), consider a new market where the price of the output at time t is given by:

$$P(t) = X(t)(1 - \eta Q(t)) \tag{3.1}$$

where *Q(t)* is the total market output, $\eta > 0$ is a constant and *X(t)* is a exogenous shock process that follows a Geometric Brownian Motion (GBM):

$$dX(t) = \mu X(t)dt + \sigma X(t)dw(t) \tag{3.2}$$

where μ is the drift rate, r is the risk-free rate, σ is the standard deviation, and dw is the increment of the Wiener process. It is assumed that $r > \mu$ in order to obtain a solution.

To gain access to the cash flows available at a particular time t, denoted by:

$$\pi\left(t\right) = \big(Q\left(t\right)P\left(t\right)\left(1-\rho\right) - c_v Q\left(t\right)\big)\left(1-\tau\right) \tag{3.3}$$

where ρ is a royalty fee, τ is a corporate tax and c_v are the costs of production per unit of Q, firms must invest in installed capacity Q. The cost per unit of installed capacity Q is given by δ, thereby the investment cost in installed capacity is given by δQ.

The model we develop assumes that there is a government and private firms. The value of the private firm is denoted by V and the value of the government by G. In this Chapter, the government only has interests in the tax that it can collect from the private firms, while the firms have interests in the cash-flows that can be obtained from being in the market. The investment opportunity that firms hold, as they are still waiting to invest, is denoted by the subscript 0.

Since we consider two scenarios, a safe one where the government commits to a tax scheme and is not threatening with expropriation, and a risky one where the government is an opportunistic agent, and will expropriate if it benefits from the extra cash-flows from operating the business itself, the superscript $i = \{s, e\}$, is used to identify each scenario, where s represents the safe scenario and e represents the one with expropriation risk (the risky one for simplicity). Also, in order to distinguish between the cash-flows that accrue to the private firms from being in the market, the subscript $j = \{m, l_m, l_d, f\}$ is used, representing the value for the monopolist, leader in monopoly, leader in duopoly and follower, respectively. In the case of the government the subscripts g_m, g_{l_m}, g_{l_d} and g_f are used, representing the cash-flows accruing from the tax collected from the monopolist, the leader while monopolist, the leader in a duopoly, and the follower, respectively.

The following assumptions are made in order to simplify the modeling problem:

Assumption 1. *The government commits to a tax scheme that the firms must comply with.*

Under this assumption, the government sets a corporate tax, denoted by τ, and also has the possibility of charging royalties, denoted by ρ, that the firm must comply with. This tax scheme is known by firms at the beginning of the process.

Assumption 2. *The firm has no outside opportunities.*

Assumption 3. *There are no informational asymmetries.*

This assumption states that all information is available to all parts involved. Under this assumption all parts involved can anticipate what others will do.

Assumption 4. *Firms produce at full capacity*

It is assumed that firms produce the quantity that they invested for, otherwise firms could slow production to artificially inflate prices.

Following the framwork of Huisman and Kort (2015), the private firm investment problem is solved as an optimal stopping problem in dynamic programming, formalized as:

$$V_j^i(X) = \max_{T \geqslant 0, Q_j^i \geqslant 0} \mathrm{E}\left[\int_{t=T}^{\infty} \pi_j^i\left(t\right) e^{-rt} dt - \delta Q_j^i e^{-rT} \middle| X(0) = X \right] \tag{3.4}$$

where T is the time when the investment is made. The process is thought to start at time 0. This assumption is made to ensure that the first observed t is inferior to the actual investment moment, T. The level of X at which firms are indifferent between investing and not investing is noted by X_j^{i*}. For values of $X < X_j^{i*}$ firms do not invest as they are still in the continuation region waiting for the market to develop.

When $X > X_j^{i*}$ it is optimal to invest immediately since the process reaches the stopping region. The optimal investment timing, T, is the first time the stochastic process X reaches X_j^{i*}. Since we are assuming that $t < T$, we have that the level of X at the initial point is too low for the investment to be made, $X < X_j^{i*}$.

To find the optimal quantity for any level of X we maximize the value of the firm at moment of investment, with respect to Q_j^i:

$$V_j^i(X) = \max_{Q_j^i \geqslant 0} \mathrm{E}\left[\int_{t=0}^{\infty} \pi_j^i(t)\, e^{-rt} dt - \delta Q_j^i \,\middle|\, X(0) = X\right] \tag{3.5}$$

from there is only a matter of incorporating the optimal quantity into the traditional real options framework to solve the problem.

Note that any time formulation uses the general subscripts i or j, it symbolizes that it will be carried out throughout the work and will be used in different contexts. The proofs for all propositions are given in the Appendix.

3.1 Monopoly

Proposition 1. *Investment option value for the safe Monopolist:*

The cash flows accruing to a monopolist, at time t, are given by:

$$\pi_m^i(t) = \left(Q_m^i(t)\, X(t) \left(1 - \eta Q_m^i(t)\right)(1-\rho) - c_v Q_m^i(t)\right)(1-\tau) \tag{3.6}$$

and the expected value of this cash flows at the moment of investment is given by:

$$\Pi_m^i\left(X, Q_m^i\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_m^i(t)\, e^{-rt} dt\right] = \left(\frac{Q_m^i X(1-\eta Q_m^i)}{r-\mu}(1-\rho) - \frac{c_v Q_m^i}{r}\right)(1-\tau) \tag{3.7}$$

The value of the safe monopolistic after investment, $X \geq X_m^{s*}$, is:

$$V_m^s(X) = \Pi_m^s\left(X, Q_m^{s*}\right) - \delta Q_m^{s*} \tag{3.8}$$

and its value before investing, as it still holds the option to invest, $X < X_m^{s*}$, is:

$$V0_m^s(X) = \left(\Pi_m^s\left(X_m^{s*}, Q_m^{s*}\right) - \delta Q_m^{s*}\right)\left(\frac{X}{X_m^{s*}}\right)^{\beta_1} \tag{3.9}$$

where:

$$X_m^{s*} = \frac{\beta_1 + 1}{\beta_1 - 1} \frac{(r-\mu)\left(c_v(1-\tau) + r\delta\right)}{r(1-\rho)(1-\tau)} \tag{3.10}$$

is the investment trigger, and:

$$Q_m^{s*} = \frac{1}{(\beta_1 + 1)\eta} \tag{3.11}$$

is the optimal capacity at the moment of investment, and:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{3.12}$$

The optimal market price for investing is given by:

$$P_m^{s*} = X_m^{s*}\left(1 - \eta Q_m^{s*}\right) \tag{3.13}$$

Since the safe monopolist, once invested, will be indefinitely in the market, the present value of the future cash flows are treated as the present value of a perpetuity (3.7), accounting for the taxes collected by the government.

For values of $X < X_m^{s*}$, the safe monopolist will hold its investment and wait for more information since the optimal time for investment, given by X_m^{s*} (3.10), is yet to be reached, and so, its value is the value of the option to invest itself, denoted by $V0_m^s(X)$ (3.9). The first moment X reaches X_m^{s*} the safe monopolist will invest δQ_m^{s*} and receive $\Pi_m^s(X, Q_m^{s*})$, and its value will be given by $V_m^s(X)$ (3.8).

By substituting (3.10) and (3.11) into (3.1) we obtain the optimal price for investment, P_m^{e*} (3.13).

Note that the results are similar to the results of Huisman and Kort (2015) in their monopolistic case. The optimal quantity at the moment of investment, Q_m^{s*} (3.11), is the same. So we have that taxes have no impact on the scale, but they do affect the timing, X_m^{s*} (3.10). In this case they make the monopolist invest later.

Proposition 2. *Value of the passive government in a monopolistic setting:*

The cash flows accruing to the passive government, in a monopolistic setting, at time t, are given by:

$$\begin{aligned}\pi_{g_m}^i(t) = &\left(Q_m^i(t)\,X(t)\left(1 - \eta Q_m^i(t)\right)\right)\rho \\ &+ \left(Q_m^i(t)\,X(t)\left(1 - \eta Q_m^i(t)\right)(1-\rho) - c_v Q_m^i(t)\right)\tau\end{aligned} \tag{3.14}$$

and the expected value of the cash flows at the moment the monopolist invests is given by:

$$\begin{aligned}\Pi_{g_m}^i(X, Q_m^i) &= E\left[\int_{t=0}^{\infty} \pi_{g_m}^i(t)e^{-rt}dt\right] \\ &= \frac{Q_m^i X(1-\eta Q_m^i)}{r-\mu}\rho + \left(\frac{Q_m^i X(1-\eta Q_m^i)}{r-\mu}(1-\rho) - \frac{c_v Q_m^i}{r}\right)\tau\end{aligned} \tag{3.15}$$

The value of the passive government after the safe monopolist invests, $X \geq X_m^{s*}$, is:

$$G_m^s(X) = \Pi_{g_m}^s\left(X, Q_M^{s*}\right) \tag{3.16}$$

and its value before the safe monopolist invests, $X < X_m^{s*}$, is:

$$G0_m^s(X) = \Pi_{g_m}^s\left(X_m^{s*}, Q_m^{s*}\right)\left(\frac{X}{X_m^{s*}}\right)^{\beta_1} \tag{3.17}$$

Before the safe monopolist invests, $X < X_m^{s*}$, the government holds a claim, noted by G_{0m}^{s} (3.17), on the safe monopolist's option, that will be exercised once the safe monopolist invests. The exercise of this option gives the government the right to the tax proceeds from the tax scheme set by the government himself. Since the safe monopolist will be in the market indefinitely, the value of the cash-flows accruing to the government, from the tax collected, are considered as the present value of a perpetuity (3.15). These cash-flows come from the percentage amount collected by the royalty fee ρ and the corporate tax τ. After the safe monopolist invests, when $X \geq X_m^{s*}$, the value of the government, noted by $G_m^s(X)$ (3.16), is the value those cash-flows.

3.2 Competitive

Following Fudenberg and Tirole, 1985 framework, the following competitive setting consists of a leader and a follower, with symmetric cost structures. The leader is the first firm to invests and the follower is the second one. Since there are no cost advantages for either firm, a "natural" leader will not emerge and it is unclear which role each firm will have, and thus, firms will have the incentive to preempt its rival in order to secure the monopolistic rents that will prevail while the follower still waits for the optimal time to invest.

The following propositions take into account such actions.

Proposition 3. *Investment option value for the safe follower:*
The cash flows accruing to the follower, at time t, are given by,

$$\pi_f^i(t) = \left(Q_f^i(t)X(t)\left(1-\eta\left(Q_l^i(t)+Q_f^i(t)\right)\right)\left(1-\rho\right)-c_vQ_f^i(t)\right)\left(1-\tau\right) \tag{3.18}$$

and the expected value of the cash flows at the moment the follower invests is given by:

$$\begin{aligned}\Pi_f^i\left(X,Q_l^i,Q_f^i\right) &= \mathrm{E}\left[\int_{t=0}^{\infty}\pi_f^i(t)\,e^{-rt}dt\right] \\ &= \left(\frac{Q_f^iX\left(1-\eta(Q_l^i+Q_f^i)\right)}{r-\mu}(1-\rho)-\frac{c_vQ_f^i}{r}\right)(1-\tau)\end{aligned} \tag{3.19}$$

The value of the safe follower after investment, $X \geq X_f^{s*}\left(Q_l^s\right)$, is given by:

$$V_f^s\left(X,Q_l^s\right) = \Pi_f^s\left(X,Q_l^s,Q_f^{s*}\left(Q_l^s\right)\right)-\delta Q_f^{s*}\left(Q_l^s\right) \tag{3.20}$$

and its value before investing, as it still holds the option to invest, $X < X_f^{s*}\left(Q_l^s\right)$, is:

$$V_{0f}^{s}\left(X,Q_l^s\right) = \left(\Pi_f^s\left(X_f^{s*}\left(Q_l^s\right),Q_l^s,Q_f^{s*}\left(Q_l^s\right)\right)-\delta Q_f^{s*}\left(Q_l^s\right)\right)\left(\frac{X}{X_f^{s*}(Q_l^s)}\right)^{\beta_1} \tag{3.21}$$

where

$$X_f^{s*}\left(Q_l^s\right) = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)\left(c_v\left(1-\tau\right)+r\delta\right)}{r\left(1-\rho\right)\left(1-\tau\right)\left(1-\eta Q_l^s\right)} \tag{3.22}$$

is the investment trigger, conditional on the quantity choice by the leader, and

$$Q_f^{s*}(Q_l^s) = \frac{1-\eta Q_l^s}{(\beta_1+1)\eta} \tag{3.23}$$

is the optimal capacity at the moment of investment, conditional on the quantity choice by the leader.

The optimal market price for investing is given by:

$$P_f^{s*}\left(Q_l^s\right) = X_f^{s*}\left(Q_l^s\right)\left(1-\eta\left(Q_l^s+Q_f^{s*}(Q_l^s)\right)\right) \tag{3.24}$$

Similar to the case of the safe monopolist, after investing, the safe follower will be in the market indefinitely, and so, the present value of the cash-flows accruing to him is considered as a perpetuity (3.19).

For values of $X < X_f^{s*}\left(Q_l^s\right)$, where $X_f^{s*}\left(Q_l^s\right)$ (3.22) is the optimal time to invest, the value of the safe follower is the value of the investment option that it holds, denoted by $V_{0f}^{s}\left(X, Q_l^s\right)$ (3.21). The first moment X reaches the trigger X_f^{s*}, the safe follower will invest $\delta Q_f^{s*}\left(Q_l^s\right)$ and will receive $\Pi_f^s\left(X_f^{s*}(Q_l^s), Q_l^s, Q_f^{s*}(Q_l^s)\right)$. Its value after investment is given by $V_f^s\left(X, Q_l^s\right)$ (3.20).

In the case of the follower, since the leader has already invested, there are no strategic actions that the follower can take to affect the leader's decision. Because this is the case, the follower will only take into consideration the leader's installed capacity and will adapt its optimal investment timing and capacity. The follower's optimal timing, $X_f^{s*}\left(Q_l^s\right)$, and optimal capacity, $Q_f^{s*}\left(Q_l^s\right)$, are both conditional on the amount capacity invested by the leader.

The moment the follower enters the market, the leader will lose its monopolistic rents and the market will become a duopoly, that may be shared in different proportions, dependent on the difference in investment made by the leader and the follower.

Proposition 4. *Investment option value for the safe leader:*

The cash flows accruing to the leader, at time t, before the follower invests, are given by:

$$\pi_{l_m}^i\left(t\right) = \left(Q_l^i\left(t\right)X\left(t\right)\left(1-\eta Q_l^i\left(t\right)\right)\left(1-\rho\right)-c_vQ_l^i\left(t\right)\right)\left(1-\tau\right) \tag{3.25}$$

and after the follower invests are given by:

$$\pi_{l_d}^i\left(t\right) = \left(Q_l^i\left(t\right)X\left(t\right)\left(1-\eta\left(Q_l^i\left(t\right)+Q_f^i\left(t\right)\right)\right)\left(1-\rho\right)-c_vQ_l^i\left(t\right)\right)\left(1-\tau\right) \tag{3.26}$$

The expected value of the cash-flows at the moment the leader invests, knowing that *the follower is yet to invest and is the next to enter the market, is given by:*

$$\Pi_{l_m}^i\left(X,Q_l^i\right)=\mathrm{E}\left[\int\limits_{t=0}^{\infty}\pi_{l_m}^i(t)\,e^{-rt}dt\right]+\mathrm{E}\left[\int\limits_{T_f^{i*}}^{\infty}\left(\pi_{l_d}^i(t)-\pi_{l_m}^i(t)\right)e^{-rt}dt\right]$$
$$=\left(\left(\frac{Q_l^iX(1-\eta Q_l^i)}{r-\mu}-\left(\frac{X}{X_f^{i*}(Q_l^i)}\right)^{\beta_1}\frac{X_f^{i*}(Q_l^i)\eta Q_f^{i*}(Q_l^i)Q_l^i}{r-\mu}\right)(1-\rho)-\frac{c_vQ_l^i}{r}\right)(1-\tau) \tag{3.27}$$

and after the follower invests, is given by:

$$\Pi_{l_d}^i\left(X,Q_l^i\right)=\mathrm{E}\left[\int\limits_{t=T_f^{i*}}^{\infty}\pi_{l_d}^i(t)\,e^{-rt}dt\right]=\left(\frac{Q_l^iX\left(1-\eta(Q_l^i+Q_f^i)\right)}{r-\mu}(1-\rho)-\frac{c_vQ_l^i}{r}\right)(1-\tau) \tag{3.28}$$

The value of the safe leader after investing, and before the safe follower invests, $X_f^{s*}\left(Q_l^{s*}\right)>X\geq X_p^{s*}$, is:

$$V_{l_m}^s(X)=\Pi_{l_m}^s\left(X,Q_l^{s*}\right)-\delta Q_l^{s*} \tag{3.29}$$

and its value before investing, as it still holds the option to invest, $X<X_p^{s*}<X_f^{s*}\left(Q_l^{s*}\right)$, is:

$$V0_l^s(X)=\left(\Pi_{l_m}^s\left(X_p^{s*},Q_l^{s*}\right)-\delta Q_l^{s*}\right)\left(\frac{X}{X_p^{s*}}\right)^{\beta_1} \tag{3.30}$$

where X_p^{s*}, the optimal time to invest under preemption, and Q_l^{s*}, the optimal capacity at the moment of investment, are numerically obtained by simultaneously solving equations:

$$V_{l_m}^s\left(X_p^{s*},Q_l^{s*}\right)=V0_f^s\left(X_p^{s*},Q_l^{s*}\right) \tag{3.31}$$

and

$$\left.\frac{\partial V_{l_m}^s\left(X,Q_l^{s*}\right)}{\partial Q_l^{s*}}\right|_{X=X_p^{s*}}=0 \tag{3.32}$$

The optimal market price for investing is given by:

$$P_l^{s*}=X_p^{s*}\left(1-\eta\left(Q_l^{s*}+Q_f^{s*}(Q_l^{s*})\right)\right) \tag{3.33}$$

Because there is an advantage in being the leader, due to the monopolistic rents that can be obtained by being alone in the market while is not optimal for the follower to invest, there is an incentive to preempt the rival firm. By preempting, firms strategically position themselves to gain from being alone in the market. Since firms are symmetric, they will try to invest a little bit earlier than their competitor to get the monopolistic rents, and thus, if firm 1 wants to invest at X, firm 2 will try to invest at $X-\varepsilon$, and firm 1 will respond by investing at $X-2\varepsilon$, and this process will continue, $X-n\varepsilon$, until a point, we denote by the subscript p, is reached where the values of being the leader or follower equal ($V_{l_m}^{i}=V_{0f}^{i}$), and there is no more incentive to try and invest sooner than the rival. At this point, for the specific case X_p^{s*} (obtained by (3.31)), firms are indifferent in being the leader or the follower, and follows that at this point, with probability $1/2$, one of the firms will invest and will be the leader.

Since the follower will eventually enter the market, the present value of the leader's monopolistic rents need a correction for the difference in the value of being a monopolist or being in a duopoly. This correction is obtained by the stochastic factor $\left(\frac{X}{X_f^{s*}(Q_l^{s*})}\right)^{\beta_1}$ in equation (3.27), that simply states that as the stochastic variable X approaches X_f^{s*}, the value of the monopolistic rents tend to the value of being a leader in a duopoly.

For values of $X_f^{s*}>X\geq X_p^{s*}$, the value of the leader, given by $V_{l_m}^{s}(X)$ (3.29), is the value of the monopolistic rents corrected for the eventual entry of the follower in the market minus the investment costs, δQ_l^{s*}, and for for values of $X<X_p^{s*}$ the value of the leader is the value of option that it holds, denoted by $V_{0l}^{s}(X)$ (3.30).

Proposition 5. *Value for the passive government in a competitive setting:*

The cash flows accruing to the passive government, from the tax collect from the leader, at time t, while the leader is a monopolist in the market, are given by:

$$\pi_{g_{l_m}}^{i}(t)=\left(Q_l^i(t)\,X(t)\left(1-\eta Q_l^i(t)\right)\right)\rho+\left(Q_l^i(t)\,X(t)\left(1-\eta Q_l^i(t)\right)(1-\rho)-c_vQ_l^i(t)\right)\tau \tag{3.34}$$

and after the follower invests, are given by:

$$\begin{aligned}\pi_{g_{l_d}}^{i}(t)=&\left(Q_l^i(t)\,X(t)\left(1-\eta\left(Q_l^i(t)+Q_f^i(t)\right)\right)\right)\rho\\&+\left(Q_l^i(t)\,X(t)\left(1-\eta\left(Q_l^i(t)+Q_f^i(t)\right)\right)(1-\rho)-c_vQ_l^i(t)\right)\tau\end{aligned} \tag{3.35}$$

The expected value of the cash flows at the moment the leader invests, and before the follower invests, is given by:

$$\begin{aligned}\Pi_{g_{l_m}}^{i}\left(X,Q_l^i\right)&=\mathrm{E}\left[\int_{t=0}^{\infty}\pi_{g_{l_m}}^{i}(t)\,e^{-rt}dt\right]+\mathrm{E}\left[\int_{T_f^{i*}}^{\infty}\left(\pi_{g_{l_d}}^{i}(t)-\pi_{g_{l_m}}^{i}(t)\right)e^{-rt}dt\right]\\&=\left(\frac{Q_l^iX(1-\eta Q_l^i)}{r-\mu}-\left(\frac{X}{X_f^{i*}(Q_l^i)}\right)^{\beta_1}\frac{X_f^{i*}(Q_l^i)\eta Q_f^{i*}(Q_l^i)Q_l^i}{r-\mu}\right)\rho\\&\quad+\left(\left(\frac{Q_l^iX(1-\eta Q_l^i)}{r-\mu}-\left(\frac{X}{X_f^{i*}(Q_l^i)}\right)^{\beta_1}\frac{X_f^{i*}(Q_l^i)\eta Q_f^{i*}(Q_l^i)Q_l^i}{r-\mu}\right)(1-\rho)-\frac{c_vQ_l^i}{r}\right)\tau\end{aligned} \tag{3.36}$$

and the expected value of the cash flows, from tax collected from the leader, at the moment the follower invests is given by:

$$\Pi^i_{g_{l_d}}\left(X, Q^i_l\right) = \mathrm{E}\left[\int\limits_{t=T^{i*}_f}^{\infty} \pi^i_{g_{l_d}}(t)\, e^{-rt} dt\right]$$
$$= \frac{Q^i_l X\left(1-\eta(Q^i_l + Q^{i*}_f(Q^i_l))\right)}{r-\mu}\rho + \left(\frac{Q^i_l X\left(1-\eta(Q^i_l + Q^{i*}_f(Q^i_l))\right)}{r-\mu}(1-\rho) - \frac{c_v Q^i_l}{r}\right)\tau \tag{3.37}$$

The value of the passive government, from the tax collected from the leader, in a duopoly, $X \geq X^{s*}_f\left(Q^{s*}_l\right)$, is given:

$$G^s_{l_d}(X) = \Pi^s_{g_{l_d}}\left(X, Q^{s*}_l\right) \tag{3.38}$$

and while the leader is a monopolist, $X^{s*}_p \geq X < X^{s*}_f\left(Q^{s*}_l\right)$, is:

$$G^s_{l_m}(X) = \Pi^s_{g_{l_m}}\left(X, Q^{s*}_l\right) \tag{3.39}$$

Before the leader invests, $X < X^{s*}_p$, the value of the government is the value of the claim that it holds on the leader's option, noted by:

$$G0^s_l(X) = \Pi^s_{g_{l_m}}\left(X^{s*}_p, Q^{s*}_l\right)\left(\frac{X}{X^{s*}_p}\right)^{\beta_1} \tag{3.40}$$

The cash flows accruing to the passive government, from the tax collected from the follower, at time t, are given by:

$$\pi^i_{g_f}(t) = \left(Q^i_f(t)\, X(t)\left(1-\eta\left(Q^i_l(t) + Q^i_f(t)\right)\right)\right)\rho$$
$$+\left(Q^i_f(t)\, X(t)\left(1-\eta\left(Q^i_l(t) + Q^i_f(t)\right)\right)(1-\rho) - c_v Q^i_f(t)\right)\tau \tag{3.41}$$

and the expected value of the cash flows, from the tax collected from the follower, at the moment the follower invests is given by:

$$\Pi^i_{g_f}\left(X, Q^i_l, Q^i_f\right) = \mathrm{E}\left[\int\limits_{t=0}^{\infty} \pi^i_{g_f}(t)\, e^{-rt} dt\right]$$
$$= \frac{Q^i_f X\left(1-\eta(Q^i_l + Q^i_f)\right)}{r-\mu}\rho + \left(\left(\frac{Q^i_f X\left(1-\eta(Q^i_l + Q^i_f)\right)}{r-\mu}\right)(1-\rho) - \frac{c_v Q^i_f}{r}\right)\tau \tag{3.42}$$

The value of the passive government, from the tax collected from the follower, after investment, $X \geq X_f^{s*}\left(Q_l^{s*}\right)$, is given:

$$G_f^s\left(X\right) = \Pi_{g_f}^s\left(X, Q_l^{s*}, Q_f^{s*}(Q_l^{s*})\right) \tag{3.43}$$

and before the follower invests, $X < X_f^{s*}\left(Q_l^{s*}\right)$, the value of the government is the value of the claim that it holds on the followers option, noted by:

$$G_{0f}^s\left(X\right) = \Pi_{g_f}^s\left(X_f^{s*}(Q_l^{s*}), Q_l^{s*}, Q_f^{s*}(Q_l^{s*})\right)\left(\frac{X}{X_f^{s*}(Q_l^{s*})}\right)^{\beta_1} \tag{3.44}$$

[1] The results for the passive government have a similar intuition to the case of the private firms. The difference being that it does not have investment costs and only collects the tax proceeds from the tax scheme in place. We have that the present value of the cash-flows from the leader, similar to the safe leader's case, need a correction for values of $X < X_f^{s*}(Q_l^{s*})$ (3.36). After the follower enters the market, $X \geq X_f^{s*}(Q_l^{s*})$, the value of the government is the value of tax collected from both firms in a duopoly, equation (3.38) for the leader and equation (3.43) for the follower. Before either of them invest, $X < X_p^{s*} < X_f^{s*}(Q_l^{s*})$, the government holds a claim on the investment options of both, leader and follower. This claim gives the government the right to the tax proceeds from the tax scheme.

3.3 Numerical Results

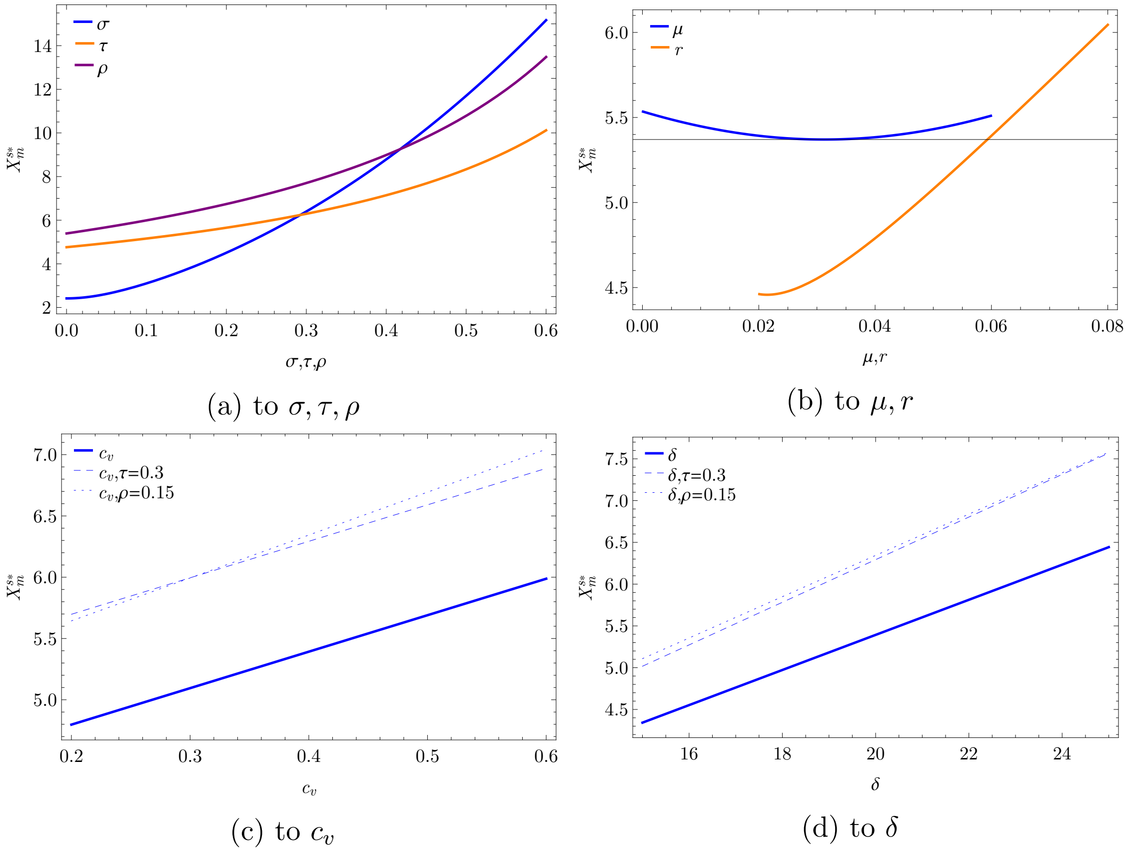
This section contains some numerical results that illustrate the propositions laid out before.

The base-case parameter values are: $r = 0.06$, $\mu = 0.02$, $\sigma = 0.25$, $c_v = 0.4$, $\delta = 20$, $\eta = 0.05$, $\rho = 0$ and $\tau = 0.15$

3.3.1 Monopoly

In the case of the investment trigger for the safe monopolist, X_m^{s*}, the results are as expected. From Figure 3.1a, we can see that as volatility increases, the safe monopolist will tend to invest later. This result simply indicates that, as the market gets to be more volatile, firms will want a bigger "buffer" to account for the larger swings that may happen to their cash-flows. Also, in graph 3.1a, as expected, higher values of the royalty fee ρ, and profits tax τ makes the monopolist invest later, simply because for higher values, firm's cash-flows would be less than what would be desirable, so firms wait longer for the market to develop, in order to obtain higher market prices to compensate for the loss from taxes. Because ρ is a direct fee on revenues, it has a higher weight in delaying the firm's investment than the corporate tax τ.

[1]Note that total value for the government, that comes from the tax collected from the leader and the follower, is the aggregate of the value that it gets from the tax collected from both. Since this work is more interested in the firm's perspective, this formulation only serves as the basis for the next Chapter and the results will not be analyzed



(a) to σ, τ, ρ

(b) to μ, r

(c) to c_v

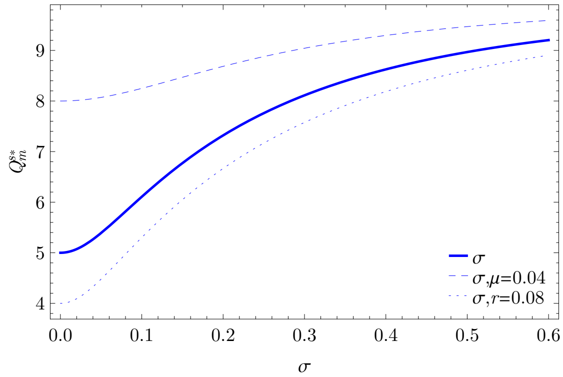
(d) to δ

Figure 3.1: Sensitivity of X_m^{s*} to its variables, moved independently

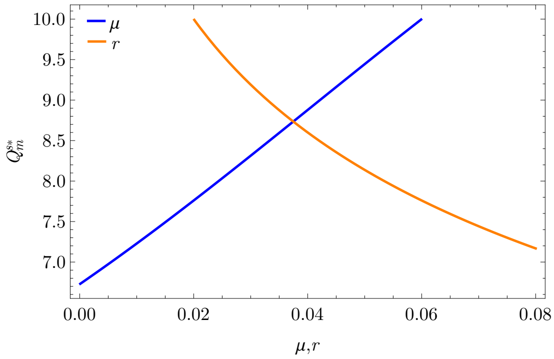
In Figure 3.1b it's possible to see that as the risk free rate, r, increases, the safe monopolist will invest later, demanding higher prices, but in the case of the growth rate, μ, is not so clear as we see a non-monotonic effect. For increasing values of μ, further from r, the effect is to decrease the investment timing, but for increasing values closer to the risk free rate the effect is to delay the investment.

In the case of the production costs, c_v (3.1c), and investment costs, δ (3.1d), the results simply show that as investment costs or production costs increase, the safe monopolist will wait for better times in the market, symbolized by higher prices, to compensate for the higher costs.

From equation (3.11) we can see that the installed capacity that the monopolist is going to invest in is only affected by η and β_1, which internally is affected by σ, μ and r. This result is similar to the one obtained by Huisman and Kort (2015). From Figure 3.2a we see that the safe monopolist will invest more as volatility increases in the market. Since, as we saw before, for higher values of σ the safe monopolist will invest later in a bigger market, the installed capacity also needs to account for the bigger market that the firm wants to capture, and that's what we see in Figure 3.2a. Figure 3.2b shows that as r increases the installed capacity that the safe monopolist is willing to invest diminishes. So we have that has r increases the safe monopolist will invest later and with less capacity. In the case of the parameter μ, the quantity to be installed increases, simply because the market has more potential for growth, but the connection between investment timing and quantity is not so clear, since for



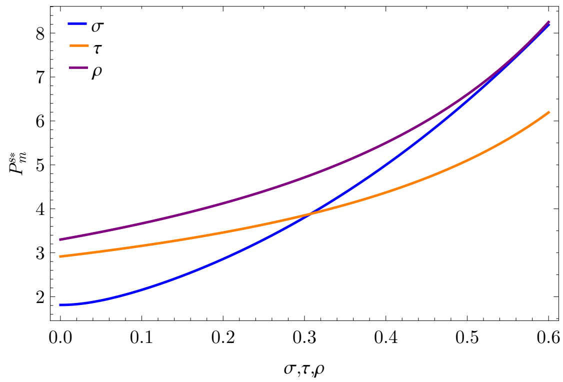
(a) to σ



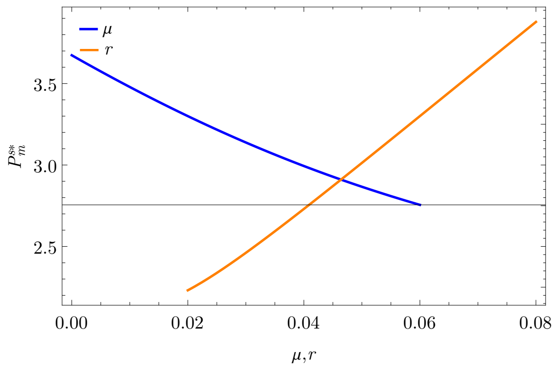
(b) to μ, r

Figure 3.2: Sensitivity of Q_m^{s*} to its variables, moved independently

increasing values of μ further from r the timing decreases and quantity increases, but for increasing values of μ closer to r the timing increases but quantity still increases.



(a) to σ

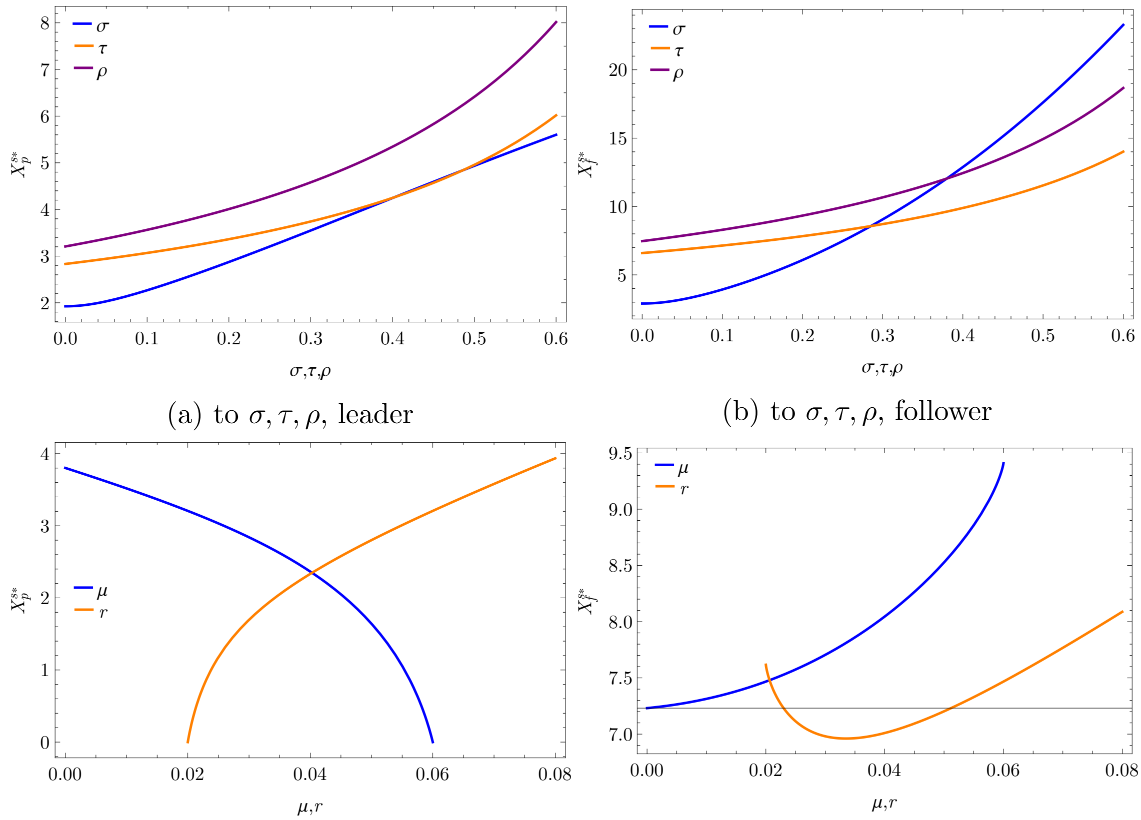


(b) to μ, r

Figure 3.3: Sensitivity of the optimal price, P_m^{s*}

Figure 3.3 shows the optimal price at which the safe monopolist will invest. The investment price is obtained by simply substituting the trigger and optimal capacity of the safe monopolist into equation (3.1). The movements in the optimal price capture both the movements in the optimal timing and optimal capacity, and give good intuition about the firm's decision. We have that as the parameters σ, τ and ρ increase, the safe monopolist will wait for higher prices (Figure 3.3a). In the case of μ and r the results are inverse. For increasing values of μ the demanded price decreases, while for r the demanded price increases (Figure 3.3b). The price movements for different values of c_v and δ are not shown as they have a straightforward intuition. If we take into account that quantity is not affected by them, only the timing of investment increases, we have that for higher of c_v and δ the optimal price for investment increases to compensate for the higher costs.

3.3.2 Competitive



(a) to σ, τ, ρ, leader

(b) to σ, τ, ρ, follower

(c) to μ, r, leader

(d) to μ, r, follower

Figure 3.4: Sensitivity of X_p^{s*}/X_f^{s*} to its variables, moved independently

From Figure's 3.4a and 3.4b, we can see that for increasing values of σ, τ and ρ, the optimal timing for the leader, X_p^{s*}, and the optimal timing for the follower, X_f^{s*}, move similarly to the case of the safe monopolist (Figure 3.1a), and have the same explanation as before.

In the case of μ, we have that as it increases, the leader will invest sooner (Figure 3.4c) which is the opposite of the follower which will invest later (Figure 3.4d). For r we have that as it increases, the leader, similar to the safe monopolist, will invest later, but the for case of the follower we have a non-monotonic effect. For increasing values of r close to μ the follower will invest later, but for increasing values of r further from μ will invest later. Bear in mind that the optimal time for the follower depends on the quantity invested by the leader, and so when we look at the optimal time to invest for the follower we have to consider the quantity invested by the leader, which is talked about bellow.

Similar to Huisman and Kort (2015), we have that, for sufficiently small values of μ, for increasing low values of σ the follower will invest more than the leader until a cross point is reached where the leader will invest the same as the follower, and after that point, the leader will invest more than the follower (Figure 3.5). But for sufficiently large values of μ, as σ increases the leader will always invest more than the follower.

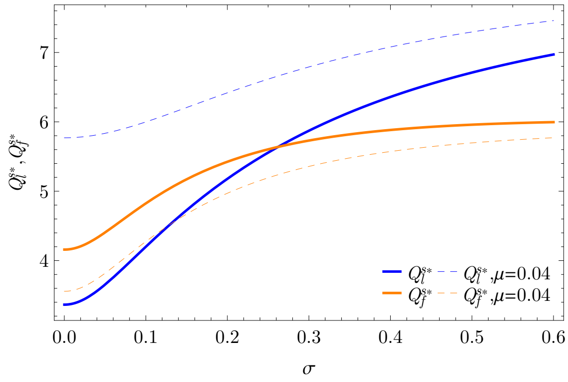
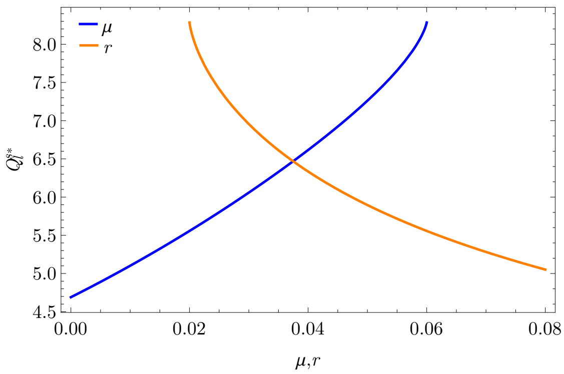
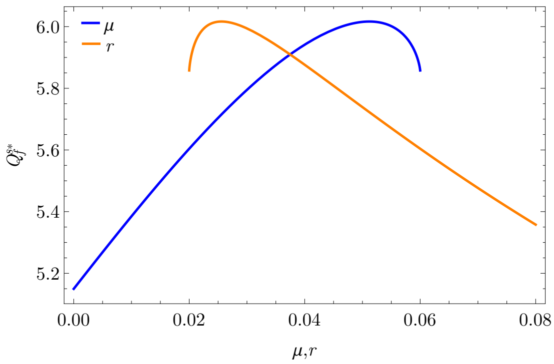


Figure 3.5: Sensitivity of Q_l^{s*}/Q_f^{s*} to σ



(a) to μ, r



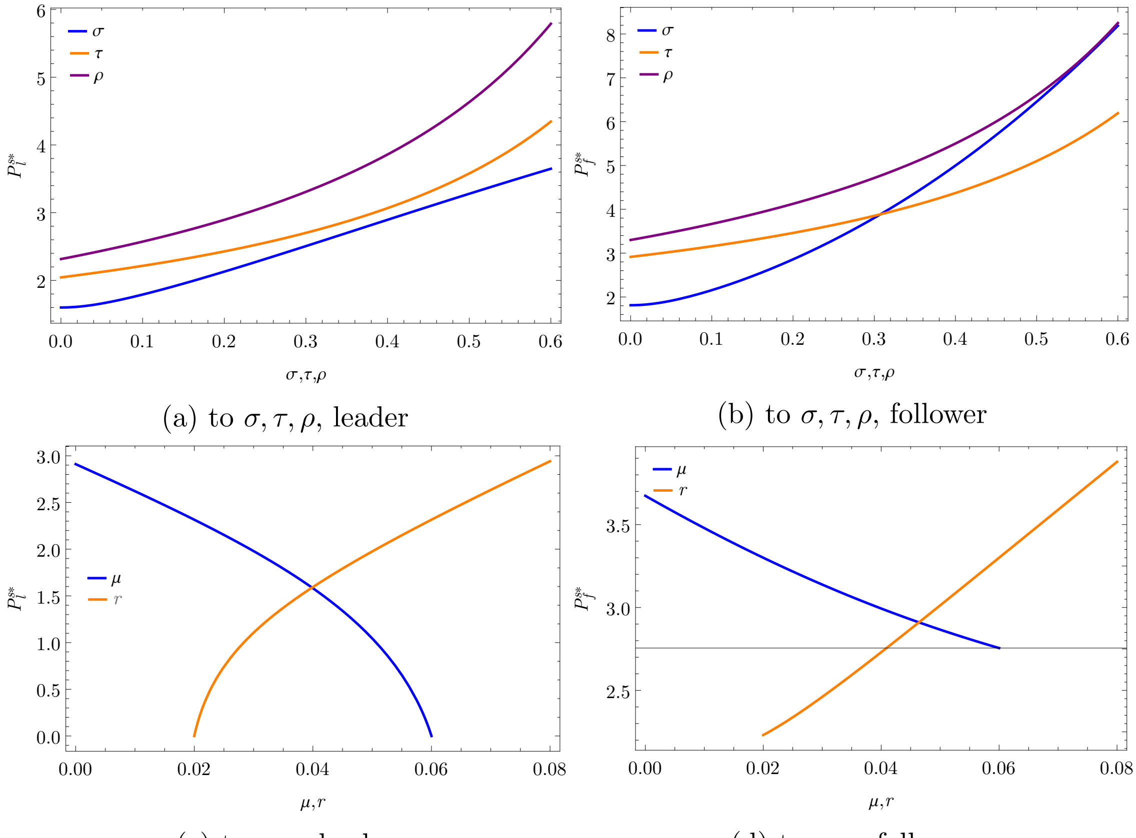
(b) to μ, r

Figure 3.6: Sensitivity of Q_l^{s*}/Q_f^{s*} to μ and r, moved independently

From Figure 3.6 we have that the amount that the leader and the follower will invest as μ increases, also increases, with the only exception being for values close to r for the follower, where a non-monotonic effect is seen. So we have that as μ increases the leader will invest sooner but with more quantity, and the follower will invest later but also with more quantity. In the case of the parameter r we have that as it increases the leader will invest less, and so does the follower, but again, in the case of the follower, for values close to μ the amount decreases.

Looking at the optimal investment prices for the leader and follower (Figure 3.7), we can see that, in both cases, as σ, τ or ρ increase the optimal price to invest also increases (Figure 3.7a and Figure 3.7b). In regards to μ and r both leader and follower have similar results. As μ increases the optimal price for both leader and follower decreases, and as r increases the optimal price for both increases. Also by comparing Figures 3.7b and 3.7d with Figures 3.3a and 3.3b of the monopolist we can observe that the price at which the follower will invest is the same as the one for the monopolist.

Figure 3.8 illustrates the market prices and market quantity for both the safe monopolist setting and competitive setting. It provides a good overview of what was pointed out previously. From Figure 3.8a we can observe that, for the same parameter values, the leader will invest sooner and at a lower price than the monopolist, while the follower will invest later but at the same price as the monopolist. We can



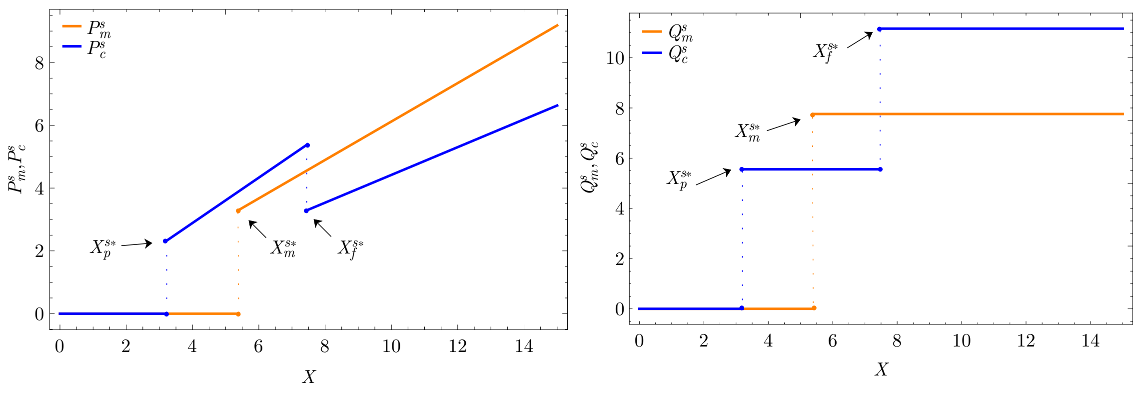
(a) to σ, τ, ρ, leader

(b) to σ, τ, ρ, follower

(c) to μ, r, leader

(d) to μ, r, follower

Figure 3.7: Sensitivity of P_l^{s*}/P_f^{s*} to its variables, moved independently



(a) Price

(b) Quantity

Figure 3.8: Market prices and quantity for the safe monopolist setting, P_m^s and Q_m^s, and safe competitive setting, P_c^s and Q_c^s, for values of X

also see that the slope of the price decreases after the follower enters the market as more quantity is being offered in the market, which is an expected result. From Figure 3.8b we can see that at moment the follower invests, which is later than the monopolist, the quantity supplied to the market will be higher than the case of the monopolist, but before the follower invests the quantity supplied in the market is actually lower than in the monopolist case.

Chapter 4

Risky Environment

This Chapter expands on the previous one and introduces the risk of expropriation that firms may face when they invest in a country. This may occur when governments have the propensity to expropriate private firm's investments if they get bigger benefits from operating the business instead of just collecting the taxes.

In this Chapter, the subscript e is used to symbolize expropriation. Note that e is used as a superscript and subscript, denoting risky environment and expropriation, respectively. Also, the parameter k and c_g are introduce. The parameter k is the fraction of the value of the business, at the moment of expropriation, that the government will compensate the private entity for the expropriation, and c_g are the costs of production of the government while operating the business.

For this Chapter some additional assumptions are made:

Assumption 5. *The government is an opportunistic agent.*

Under this assumption, following Restrepo Ochoa et al. (2015), expropriation by the opportunistic government is a reaction to high real prices of a product or service. It simply states that the government will expropriate if the price of a product or service reaches a high enough level that is optimal for the government to expropriate.

Assumption 6. *The government is less cost-efficient than a private firm.*

There is a body of literature that suggests that the government is less efficient than the private sector (e.g. Dewenter and Malatesta, 2001; Shleifer, 1998). A justification generally used is that when an enterprise is run by political institutions there is the possibility that its resources will be used to forward political ideologies, negating its efficiency (Shleifer, 1998);

Assumption 7. *Firms do not invest if the government is in the market.*

When the government is in the market it produces doubts about its intentions that can deter private firms from investing. Also, government enterprises have unfair advantages such as easier assess to credit, exception from bankruptcy, tax exemptions, direct subsidies to name a few.

4.1 Monopoly

Proposition 6. *Governments option to expropriate the risky monopolist:*
The cash flows accruing to the active government operating the monopolist's business, at time t, are given by:

$$\pi_{e_m}(t) = Q_m^e(t)\, X(t) \left(1 - \eta Q_m^e(t)\right) - c_g Q_m^e(t) \tag{4.1}$$

The expected value of the cash flows, from operating the monopolist's business, at the moment the active government expropriates the business from the monopolist, is given by:

$$\Pi_{e_m}\left(X, Q_m^e\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{e_m}(t)\, e^{-rt} dt\right] = \frac{Q_m^e X(1 - \eta Q_m^e)}{r - \mu} - \frac{c_g Q_m^e}{r} \tag{4.2}$$

The value of the government at the moment of expropriation, $X = X^*_{e_m}(Q_m^e)$, is given by:

$$G_{e_m}\left(Q_m^e\right) = \Pi_{e_m}\left(X^*_{e_m}(Q_m^e), Q_m^e\right) - k\Pi_m^e\left(X^*_{e_m}(Q_m^e), Q_m^e\right) \tag{4.3}$$

where $k \in [0, 1]$, is the fraction of the value of the monopolist's cash-flows by which the government will compensate the monopolist for the expropriation.

The value of the government after the risky monopolist invests, and before expropriation, $X^*_{e_m}(Q_m^e) > X \geq X_m^{e*}$, is given by:

$$G_m^e\left(X, Q_m^e\right) = \Pi_{g_m}^e\left(X, Q_m^e\right) + \Big(\Pi_{e_m}\left(X^*_{e_m}(Q_m^e), Q_m^e\right) - \Pi_{g_m}^e\left(X^*_{e_m}(Q_m^e), Q_m^e\right) \\ - k\Pi_m^e\left(X^*_{e_m}(Q_m^e), Q_m^e\right)\Big)\left(\frac{X}{X^*_{e_m}(Q_m^e)}\right)^{\beta_1} \tag{4.4}$$

where the trigger for expropriation, $X^*_{e_m}$, is given by:

$$X^*_{e_m}\left(Q_m^e\right) = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r - \mu)\left(c_v k + c_v (1 - k)\tau - c_g\right)}{r(1 - \rho)(1 - \tau)(1 - k)(Q_m^e \eta - 1)} \tag{4.5}$$

and X_m^{e*} is investment trigger of the risky monopolist that is yet to be defined.

The optimal market price to expropriate the monopolist is given by:

$$P^*_{e_m}\left(Q_m^e\right) = X^*_{e_m}\left(Q_m^e\right)\left(1 - \eta Q_m^e\right) \tag{4.6}$$

At the moment of expropriation, the first time X reaches $X^*_{e_m}\left(Q_m^e\right)$, the value of the government, noted by $G_{e_m}\left(Q_m^e\right)$ (4.3), is the value of the cash flows from operating the business (4.2) subtracted by the amount of compensation given to the risky monopolist. The government will loose $\Pi_{g_m}^e\left(X^*_{e_m}(Q_m^e), Q_m^e\right)$, the value from the tax collected, pay $k\Pi_m^e\left(X^*_{e_m}(Q_m^e), Q_m^e\right)$, the fraction amount of the value of the monopolist, and receives $\Pi_{e_m}\left(X^*_{e_m}(Q_m^e), Q_m^e\right)$, the value of the cash-flows from operating the business. The compensation is calculated as being a fraction of the value of the business operated by the risky monopolist at the moment of expropriation. k is an exogenous value, that is considered to be known at the beginning of the process. This value can be thought to be the traditional amount of the fraction of the value of a business that an expropriating government tends to pay as compensation for the expropriation.

After the risky monopolist invests, and before expropriation, $X^*_{e_m}(Q^e_m) > X \geq X^{e*}_m$, the value of the government is given by $G^e_m(X, Q^e_m)$ (4.4). The first term is the value of the taxes that the government collects from the monopolist, before expropriation, and the terms in parenthesis represent the correction to the value of cash-flows, that will occur once the government expropriates.

Because there is a period between the monopolist investing and the government expropriating, $X^*_{e_m}(Q^e_m) > X \geq X^{e*}_m$, the stochastic discount factor $\left(\frac{X}{X^*_{e_m}(Q^e_m)}\right)^{\beta_1}$ in (4.4) is used to correct the present value of the government for this period.

Proposition 7. *Investment option value for the risky monopolist:*

At the moment of expropriation, $X = X^*_{e_m}\left(Q^{e*}_m\right) > X^{e*}_m$, the monopolist loses its business and receives a fraction (k) of the value of the business at expropriation. At that moment, knowing that the investment has already occur, the value of the risky monopolist is given by:

$$V_{e_m} = k\Pi^e_m\left(X^*_{e_m}(Q^{e*}_m), Q^{e*}_m\right) - \delta Q^{e*}_m \tag{4.7}$$

and the value of the risky monopolist after investment, and before expropriation, $X^*_{e_m}\left(Q^{e*}_m\right) > X \geq X^{e*}_m$, is given by:

$$V^e_m(X) = \Pi^e_m\left(X, Q^{e*}_m\right)\left(1 - \left(\frac{X}{X^*_{e_m}(Q^{e*}_m)}\right)^{\beta_1 - 1}\right) + k\Pi^e_m\left(X^*_{e_m}(Q^{e*}_m), Q^{e*}_m\right)\left(\frac{X}{X^*_{e_m}(Q^{e*}_m)}\right)^{\beta_1} - \delta Q^{e*}_m \tag{4.8}$$

Before the risky monopolist invests, as it still holds the option to invest, $X < X^{e*}_m < X^*_{e_m}\left(Q^{e*}_m\right)$, its value is given by:

$$V0^e_m(X) = V^e_m\left(X^{e*}_m\right)\left(\frac{X}{X^{e*}_m}\right)^{\beta_1} \tag{4.9}$$

where X^{e*}_m, the optimal time for investment, and Q^{e*}_m, the optimal quantity at the moment of investment, are numerically obtained by simultaneous solving the smooth pasting condition (4.10) and capacity optimization condition (4.11):

$$\left.\frac{\partial V0^e_m(X, Q^{e*}_m)}{\partial X}\right|_{X = X^{e*}_m} = \left.\frac{\partial V^e_m(X, Q^{e*}_m)}{\partial X}\right|_{X = X^{e*}_m} \tag{4.10}$$

$$\left.\frac{\partial V^e_m(X, Q^{e*}_m)}{\partial Q^{e*}_m}\right|_{X = X^{e*}_m} = 0 \tag{4.11}$$

The optimal market price for investing is given by:

$$P^{e*}_m = X^{e*}_m\left(1 - \eta Q^{e*}_m\right) \tag{4.12}$$

The value of the risky monopolist after investment, for values of moment $X \geq X_m^{e*}$, is given by $V_m^e(X)$ (4.8). To receive the present value of the cash-flows from operating the business, $\Pi_m^e\left(X, Q_m^{e*}\right)$, the risky monopolist invests δQ_m^{e*}. Because eventually the monopolist will be expropriated by the government, the value of the cash-flows accruing to the risky monopolist, $\Pi_m^e\left(X, Q_m^{e*}\right)$, need a correction, as at the moment of expropriation they will be worth zero to the firm. Basically we are talking about a stream of cash-flows with expiration at the moment the stochastic process reaches $X_{e_m}^*$. The stochastic discount factor term $\left(\frac{X}{X_{e_m}^*(Q_m^{e*})}\right)^{\beta-1}$ accounts for this correction. As X approaches $X_{e_m}^*\left(Q_m^{e*}\right)$, the first term in (4.8) will tend to zero. Similarly, the compensation from the expropriation, $k\Pi_m^e\left(X_{e_m}^*(Q_m^{e*}), Q_m^{e*}\right)$ is discounted to the present by $\left(\frac{X}{X_{e_m}^*(Q_m^{e*})}\right)^{\beta_1}$. At expropriation, the first time X reaches $X_{e_m}^*(Q_m^{e*})$, the monopolist loses $\Pi_m^e\left(X_{e_m}^*(Q_m^{e*}), Q_m^{e*}\right)$ and receives $k\Pi_m^e\left(X_{e_m}^*(Q_m^{e*}), Q_m^{e*}\right)$. Equation (4.7) represents the value of the risky monopolist at expropriation.

Before investing, the value of the risky monopolist, $V_{0m}^e(X)$(4.9), is the value of the option to invest that the risky monopolist holds, that will be exercised the first time X reaches X_m^{e*}. Once exercised the value of the risky monopolist will turn into (4.8).

4.2 Competitive

Proposition 8. *Governments option to expropriate the risky follower:*

The cash flows accruing to the active government operating the follower's business, in a competitive setting, at time t, are given by:

$$\pi_{e_f}(t) = Q_f^e(t)X(t)\left(1 - \eta\left(Q_l^e(t) + Q_f^e(t)\right)\right) - c_g Q_f^e(t) \tag{4.13}$$

The expected value of the cash flows, from operating the follower's business, at the moment the active government expropriates the business from the follower, is given by:

$$\Pi_{e_f}\left(X, Q_l^e, Q_f^e\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{e_f}(t)\, e^{-rt} dt\right] = \frac{Q_f^e X\left(1 - \eta(Q_l^e + Q_f^e)\right)}{r - \mu} - \frac{c_g Q_f^e}{r} \tag{4.14}$$

The value of the government at the moment of expropriation, $X = X_{e_f}^*\left(Q_l^e, Q_f^e\right)$, is given by:

$$G_{e_f}\left(Q_l^e, Q_f^e,\right) = \Pi_{e_f}\left(X_{e_m}^*(Q_l^e, Q_f^e), Q_l^e, Q_f^e\right) - k\Pi_f^e\left(X_{e_m}^*(Q_l^e, Q_f^e), Q_l^e, Q_f^e\right) \tag{4.15}$$

and its value after the risky follower invests, before expropriation, as it still holds *the option to expropriate,* $X^*_{e_f}\left(Q^e_l, Q^e_f\right) > X \geq X^{e*}_f$, *is:*

$$G^e_f\left(X, Q^e_l, Q^e_f,\right) = \Pi^e_{g_f}\left(X, Q^e_l, Q^e_f\right) + \left(\Pi_{e_f}\left(X^*_{e_f}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right)\right.$$
$$\left.-\Pi^e_{g_f}\left(X^*_{e_f}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right) - k\Pi^e_f\left(X^*_{e_f}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right)\right)\left(\frac{X}{X^*_{e_f}(Q^e_l, Q^e_f)}\right)^{\beta_1} \tag{4.16}$$

where

$$X^*_{e_f}\left(Q^e_l, Q^e_f\right) = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu)\left(c_v k + c_v(1-k)\tau - c_g\right)}{r(1-\rho)(1-\tau)(1-k)\left((Q^e_l + Q^e_f)\eta - 1\right)} \tag{4.17}$$

is the optimal time to expropriate the follower and X^{e*}_f, *the investment trigger for the risky follower, is yet to be defined.*

The optimal market price to expropriate the follower is given by:

$$P^*_{e_f}\left(Q^e_l, Q^e_f\right) = X^*_{e_f}\left(Q^e_l, Q^e_f\right)\left(1 - \eta(Q^e_l, Q^e_f)\right) \tag{4.18}$$

The value of the active government after the risky follower invests and before expropriation, $X^*_{e_f}(Q^e_l, Q^e_f) > X \geq X^{e*}_f$, is given by $G^e_f(X, Q^e_l, Q^e_f)$ (4.16). The first argument of the equation represents the value tax proceeds that the government will collect. The terms in parenthesis represent correction that will take place to the value of the cash-flows of the government once expropriation happens, at the first time X reaches $X^*_{e_f}(Q^e_l, Q^e_f)$. At $X^*_{e_f}(Q^e_l, Q^e_f)$ the government will exercise its option to expropriate and will receive the value of the cash-flows from operating the follower's business, $\Pi_{e_f}\left(X^*_{e_m}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right)$, will pay the compensation to the risky follower, $k\Pi^e_f\left(X^*_{e_f}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right)$, and looses the value of the proceeds from the tax collected from the follower, $\Pi^e_{g_f}\left(X^*_{e_f}(Q^e_l, Q^e_f), Q^e_l, Q^e_f\right)$. The term $\left(\frac{X}{X^*_{e_f}(Q^e_l, Q^e_f)}\right)^{\beta_1}$ represents the stochastic discount factor that accounts for the correction. Its value at expropriation is given by $G_{e_f}(Q^e_l, Q^e_f)$(4.15)

Proposition 9. *Investment option value for the risky Follower:*

The value of the risky follower at the moment the government expropriates him, $X = X^*_{e_f}\left(Q^e_l, Q^{e*}_f(Q^e_l)\right) > X^{e*}_f\left(Q^e_l\right)$, *is given by:*

$$V_{e_f}\left(Q^e_l\right) = k\Pi^e_f\left(X^*_{e_f}\left(Q^e_l, Q^{e*}_f(Q^e_l)\right), Q^e_l, Q^{e*}_f(Q^e_l)\right) - \delta Q^{e*}_f(Q^e_l) \tag{4.19}$$

The value of the risky follower after investment, and before expropriation, $X^*_{e_f}\left(Q^e_l, Q^{e*}_f(Q^e_l)\right) >$

$X \geq X_f^{e*}\left(Q_l^e\right)$, *is given by:*

$$V_f^e\left(X, Q_l^e\right) = \Pi_f^e\left(X, Q_l^e, Q_f^{e*}(Q_l^e)\right)\left(1 - \left(\frac{X}{X_{e_f}^*(Q_l^e, Q_f^{e*}(Q_l^e))}\right)^{\beta_1 - 1}\right)$$
$$+ k\Pi_f^e\left(X_{e_f}^*\left(Q_l^e, Q_f^{e*}(Q_l^e)\right), Q_l^e, Q_f^{e*}(Q_l^e)\right)\left(\frac{X}{X_{e_f}^*(Q_l^e, Q_f^{e*}(Q_l^e))}\right)^{\beta_1} - \delta Q_f^{e*}(Q_l^e) \quad (4.20)$$

and before the risky follower invests, as it still holds the option to invest, $X < X_f^{e*}\left(Q_l^e\right) < X_{e_f}^*\left(Q_l^e, Q_f^{e*}(Q_l^e)\right)$, *is given by:*

$$V_{0f}^e\left(X, Q_l^e\right) = V_f^e\left(X_f^{e*}(Q_l^e), Q_l^e\right)\left(\frac{X}{X_f^{e*}(Q_l^e)}\right)^{\beta_1} \quad (4.21)$$

where $X_f^{e*}\left(Q_l^e\right)$, *the optimal time to invest, and* $Q_f^{e*}\left(Q_l^e\right)$, *the optimal quantity at the moment of investment, are numerically obtained by simultaneous solving the smooth pasting condition (4.22) and capacity optimization condition (4.23):*

$$\left.\frac{\partial V_{0f}^e(X, Q_l^e, Q_f^{e*})}{\partial X}\right|_{X = X_f^{e*}} = \left.\frac{\partial V_f^e(X, Q_l^e, Q_f^{e*})}{\partial X}\right|_{X = X_f^{e*}} \quad (4.22)$$

$$\left.\frac{\partial V_f^e(X, Q_l^e, Q_f^{e*})}{\partial Q_f^{e*}}\right|_{X = X_f^{e*}} = 0 \quad (4.23)$$

The optimal market price for investing is given by:

$$P_f^{e*}\left(Q_l^e\right) = X_f^{e*}\left(Q_l^e\right)\left(1 - \eta\left(Q_l^e + Q_f^{e*}(Q_l^e)\right)\right) \quad (4.24)$$

Since the risky follower, after investing, will eventually be expropriated, its value after investing, $X \geq X_f^{e*}\left(Q_l^e\right)$, given by $V_f^e\left(X, Q_l^e\right)$ (4.20), much like in the case of the risky monopolist, needs a correction in the value of the cash flows from operating the business. The correction is obtained in the same way as in the case of the risky monopolist, which means that the expected value from operating the business will tend to zero, and the value of the compensation will tend to its nominal value as X approaches $X_{e_f}^*$. At the moment of expropriation, the first time X reaches $X_{e_f}^*$, the value of the risky follower, given by $V_{e_f}\left(Q_l^e\right)$ (4.19), is the fraction k of the value of the business that the government compensates the firm for the the expropriation. Before investing, for values of $X < X_f^{e*}\left(Q_l^e\right)$, the value of the risky follower, given by $V_{0f}^e\left(X, Q_l^e\right)$ (4.21) is the value of the investment option that it holds.

Proposition 10. *Government's option to expropriate the risky leader:*

For values of $X_{e_{l_m}}^* < X_f^{e*}$

The cash flows accruing to the active government operating the leader's business, while the follower is yet to invest, at time t, are given by:

$$\pi_{e_{l_m}}(t) = Q_l^e(t)\, X(t) \left(1 - \eta Q_l^e(t)\right) - c_g Q_l^e(t) \tag{4.25}$$

and the cash flows accruing to the active government operating the leader's business, after the follower invests, at time t, are given by:

$$\pi_{e_{l_d}}(t) = Q_l^e(t)\, X(t) \left(1 - \eta \left(Q_l^e(t) + Q_f^e(t)\right)\right) - c_g Q_l^e(t) \tag{4.26}$$

The expected value of the cash flows, from operating the leader's business while monopolist, at the moment the active government expropriates the business from the leader, is given by:

$$\begin{aligned}\Pi_{e_{l_m}}\left(X, Q_l^e\right) &= \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{e_{l_m}}(t)\, e^{-rt} dt\right] + \mathrm{E}\left[\int_{T_f^*}^{\infty} \left(\pi_{e_{l_d}}(t) - \pi_{e_{l_m}}(t)\right) e^{-rt} dt\right] \\ &= \frac{Q_l^e X \left(1 - \eta Q_l^e\right)}{r - \mu} - \left(\frac{X}{X_f^{e*}(Q_l^e)}\right)^{\beta_1} \frac{X_f^{e*}(Q_l^e)\eta Q_f^{e*}(Q_l^e) Q_l^e}{r - \mu} - \frac{c_g Q_l^e}{r}\end{aligned} \tag{4.27}$$

The value of the government at the moment of expropriation, $X = X^*_{e_{l_m}}(Q_l^e)$, is given by:

$$G_{e_{l_m}}\left(Q_l^e\right) = \Pi_{e_{l_m}}\left(X^*_{e_{l_m}}(Q_l^e), Q_l^e\right) - k\Pi^e_{l_m}\left(X^*_{e_{l_m}}(Q_l^e), Q_l^e\right) \tag{4.28}$$

By solving the value matching condition (4.29) and numerically solving the smooth pasting condition (4.30):

$$G^e_{l_m}\left(X^*_{e_{l_m}}, Q_l^e\right) = G_{e_{l_m}}\left(X^*_{e_{l_m}}, Q_l^e\right) \tag{4.29}$$

$$\left.\frac{\partial G^e_{l_m}\left(X, Q_l^e\right)}{\partial X}\right|_{X = X^*_{e_{l_m}}} = \left.\frac{\partial G_{e_{l_m}}\left(X, Q_l^e\right)}{\partial X}\right|_{X = X^*_{e_{l_m}}} \tag{4.30}$$

we get the trigger to expropriate the risky leader, while monopolist, $X^*_{e_{l_m}}(Q_l^e)$, and the value for the government, after the risky leader invests, and before expropriation, $X^*_{e_{l_m}}\left(Q_l^e\right) > X \geq X_p^{e*}$:

$$\begin{aligned}G^e_{l_m}\left(X, Q_l^e\right) = \Pi^e_{g_{l_m}}\left(X, Q_l^e\right) + \Bigg(\Pi_{e_{l_m}}\left(X^*_{e_{l_m}}(Q_l^e), Q_l^e\right) - \Pi^e_{g_{l_m}}\left(X^*_{e_{l_m}}(Q_l^e), Q_l^e\right) \\ - k\Pi^e_{l_m}\left(X^*_{e_{l_m}}(Q_l^e), Q_l^e\right)\Bigg) \left(\frac{X}{X^*_{e_{l_m}}\left(Q_l^e\right)}\right)^{\beta_1}\end{aligned} \tag{4.31}$$

where X_p^{e*}, the investment trigger for the risky leader, is yet to be define.

For values of $X^*_{e_{l_m}} > X^{e*}_f$

The expected value of the cash flows, from operating the leader's business in a duopoly, at the moment the active government expropriates the business from the leader, is given by:

$$\Pi_{e_{l_d}}\left(X, Q^e_l\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{e_{l_d}}(t) e^{-rt} dt\right] = \frac{Q^e_l X \left(1 - \eta\left(Q^e_l + Q^{e*}_f(Q^e_l)\right)\right)}{r - \mu} - \frac{c_g Q^e_l}{r} \quad (4.32)$$

The value of the government at the moment it expropriates the leader, already in a duopoly, $X^{e*}_p < X^{e*}_f\left(Q^e_l\right) < X = X^*_{e_{l_d}}\left(Q^e_l\right)$*, is given by:*

$$G_{e_{l_d}}\left(Q^e_l\right) = \Pi_{e_{l_d}}\left(X^*_{e_{l_d}}(Q^e_l), Q^e_l\right) - k\Pi^e_{l_d}\left(X^*_{e_{l_d}}(Q^e_l), Q^e_l\right) \quad (4.33)$$

and after the follower invests, before the expropriation of the risky leader, $X^{e*}_p < X^{e*}_f\left(Q^e_l\right) \leq X < X^*_{e_{l_d}}\left(Q^e_l\right)$*, is given by:*

$$\begin{aligned} G^e_{l_d}\left(X, Q^e_l\right) = \Pi^e_{g_{l_d}}\left(X, Q^e_l\right) + \Big(\Pi_{e_{l_d}}\left(X^*_{e_{l_d}}(Q^e_l), Q^e_l\right) - \Pi^e_{g_{l_d}}\left(X^*_{e_{l_d}}(Q^e_l), Q^e_l\right) \\ -k\Pi^e_{l_d}\left(X^*_{e_{l_d}}(Q^e_l), Q^e_l\right)\Big)\left(\frac{X}{X^*_{e_{l_d}}(Q^e_l)}\right)^{\beta_1} \end{aligned} \quad (4.34)$$

where the trigger to expropriate the leader in duopoly is given by:

$$X^*_{e_{l_d}}\left(Q^e_l\right) = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu)\left(c_v k + c_v (1 - k)\tau - c_g\right)}{r(1 - \rho)(1 - \tau)(1 - k)\left((Q^e_l + Q^{e*}_f(Q^e_l))\eta - 1\right)} \quad (4.35)$$

The optimal market price to expropriate the leader in a duopoly is given by:

$$P^*_{e_{l_d}}\left(Q^e_l\right) = X^*_{e_{l_d}}\left(Q^e_l\right)\left(1 - \eta\left(Q^e_l + Q^{e*}_f(Q^e_l)\right)\right) \quad (4.36)$$

There are two cases that need to be analyzed in the case of government expropriating the leader.

The first case is for values of $X^*_{e_{l_m}} \leq X^*_{e_f}$, where $X^*_{e_{l_m}}$ denotes the trigger to expropriate the leader while monopolist. In this range, the government will expropriate the leader before the follower enters the market since the government's trigger to expropriate the leader, $X^*_{e_{l_m}}$, is at a lower level than the investment trigger for the follower, X^{e*}_f. By expropriating the leader, the government will receive the remaining value of the monopolistic rents that still exist, since the follower is yet to invest at its optimal time, plus the value that will be generated from being in a duopoly once the follower finally invests. Since this is the case, the expected value of the cash-flows accruing to the government from operating the leader's business, while being a monopolist in the market, given by $\Pi_{e_{l_m}}\left(X, Q^e_l\right)$ (4.27), need a negative correction in order to account for the arrival of the follower. The value of the government at the moment of expropriation, $X = X^*_{e_{l_m}}\left(Q^e_l\right)$, given by $G_{e_{l_m}}\left(Q^e_l\right)$ (4.28), is the value of these cash-flows subtracted by the fraction of the value that the government will compensate the leader. Because the government knows that eventually the follower will enter the market, the value that it attributes to the leader's business is the value already corrected for that fact, in this case denoted by $\Pi^e_{l_m}\left(X^*_{e_{l_m}}, Q^e_l\right)$ (3.27).

Before expropriating the leader, while monopolist, for values of $X^*_{e_{l_m}} > X \geq X^{e*}_p$, where X^{e*}_p will denote the optimal time to invest for the leader, the value of the government, noted by $G^e_{l_m}\left(X, Q^e_l\right)$ (4.31), its the present value of the tax proceeds that it gets from the leader plus the option to expropriate the leader while monopolist, the first term and second terms in equation (4.31), respectively. The first moment X hits $X^*_{e_{l_m}}$, the government exercises its option. By doing that the government loses the value of tax proceeds that it could have collected from the leader and receives the value of the cash flows from operating the leader's business.

The second case is for values of $X^*_{e_{l_m}} > X^{e*}_f$. In this range, since the trigger to expropriate the leader, while monopolist, is bigger than the follower's investment trigger. The follower will invest sooner than the optimal time for the government to expropriate the leader, while alone in the market. Since that is the case, the government will allow both firms to enter the market and will wait for the price to rise enough so it can expropriate both firms at the same time to become the monopolist in the market. Note that the trigger to expropriate the leader, while in a duopoly, equation (4.35), and the trigger to expropriate the follower, equation (4.17), are the same. Of course this only happens because we are assuming that firms are symmetric, otherwise, the triggers to expropriate would deviate.

Proposition 11. *Investment option value for the risky Leader:*

For values of $X^*_{e_{l_m}} < X^{e*}_f$

The value of the risky leader at the moment the government expropriates him, $X^{e*}_p < X = X^*_{e_{l_m}}\left(Q^{e*}_l\right)$, *is given by:*

$$V_{e_{l_m}} = k\Pi^e_{l_m}\left(X^*_{e_{l_m}}(Q^{e*}_l), Q^{e*}_l\right) - \delta Q^{e*}_l \tag{4.37}$$

The value of the risky leader after investment, and before being expropriated, $X^*_{e_{l_m}}\left(Q^{e*}_l\right) > X \geq X^{e*}_p$, *is given by:*

$$V^e_{l_m}(X) = \Pi^e_m(X, Q^{e*}_l)\left(1 - \left(\frac{X}{X^*_{e_{l_m}}(Q^{e*}_l)}\right)^{\beta_1 - 1}\right) + k\Pi^e_{l_m}(X^*_{e_{l_m}}(Q^{e*}_l), Q^{e*}_l)\left(\frac{X}{X^*_{e_{l_m}}(Q^{e*}_l)}\right)^{\beta_1} - \delta Q^{e*}_l \tag{4.38}$$

and before investing, as it still holds the option to invest, $X < X^{e*}_p < X^*_{e_{l_m}}\left(Q^e_l\right)$, *is given by:*

$$V0^e_l(X) = V^e_{l_m}(X^{e*}_p)\left(\frac{X}{X^{e*}_p}\right)^{\beta_1} \tag{4.39}$$

where X_p^{e*} and Q_l^{e*} are numerically obtained by simultaneous solving equations:

$$V_{l_m}^e\left(X_p^{e*},Q_l^{e*}\right) = V0_f^e\left(X_p^{e*},Q_l^{e*}\right) \tag{4.40}$$

and

$$\left.\frac{\partial V_{l_m}^e\left(X,Q_l^{e*}\right)}{\partial Q_l^{e*}}\right|_{X=X_p^{e*}} = 0 \tag{4.41}$$

For values of $X_{e_{l_m}}^* > X_f^{e*}$

The value of the risky leader at the moment the government expropriates him, $X = X_{e_{l_d}}^*\left(Q_l^e\right) > X_p^{e*}$, is given by:

$$V_{e_{l_d}} = k\Pi_{l_d}^e\left(X_{e_{l_d}}^*\left(Q_l^{e*}\right),Q_l^{e*}\right) - \delta Q_l^{e*} \tag{4.42}$$

and its value after the risky follower invests, and before being expropriated, $X_{e_{l_d}}^*\left(Q_l^{e*}\right) > X \geq X_f^{e*}\left(Q_l^{e*}\right) > X_p^{e*}\left(Q_l^{e*}\right)$, is given by:

$$\begin{aligned} V_{l_d}^e(X) = \Pi_{l_d}^e\left(X,Q_l^{e*}\right)&\left(1-\left(\frac{X}{X_{e_{l_d}}^*\left(Q_l^{e*}\right)}\right)^{\beta_1-1}\right) \\ &+ k\Pi_{l_d}^e\left(X_{e_{l_d}}^*\left(Q_l^{e*}\right),Q_l^{e*}\right)\left(\frac{X}{X_{e_{l_d}}^*\left(Q_l^{e*}\right)}\right)^{\beta_1} - \delta Q_l^{e*} \end{aligned} \tag{4.43}$$

After investment, $X_{e_{l_d}}^*\left(Q_l^{e*}\right) > X_f^{e*}\left(Q_l^{e*}\right) > X \geq X_p^{e*}\left(Q_l^{e*}\right)$, before the follower enters the market, its value is given by:

$$\begin{aligned} V_{l_m}^e(X) = \Pi_m^e\left(X,Q_l^{e*}\right) + &\left(\Pi_{l_d}^e\left(X_f^{e*}(Q_l^{e*}),Q_l^{e*}\right)\left(1-\left(\frac{X_f^{e*}\left(Q_l^{e*}\right)}{X_{e_{l_d}}^*\left(Q_l^{e*}\right)}\right)^{\beta_1-1}\right)\right. \\ &\left. + k\Pi_{l_d}^e\left(X_{e_{l_d}}^*\left(Q_l^{e*}\right),Q_l^{e*}\right)\left(\frac{X_f^{e*}(Q_l^{e*})}{X_{e_{l_d}}^*\left(Q_l^{e*}\right)}\right)^{\beta_1} - \Pi_m^e\left(X_f^{e*}(Q_l^{e*}),Q_l^{e*}\right)\right)\left(\frac{X}{X_f^{e*}(Q_l^{e*})}\right)^{\beta_1} \\ &- \delta Q_l^{e*} \end{aligned} \tag{4.44}$$

and before investment, $X < X_p^{e*}\left(Q_l^{e*}\right) < X_f^{e*}\left(Q_l^{e*}\right) < X_{e_{l_d}}^*\left(Q_l^{e*}\right)$, as it still holds the option to invest, is given by:

$$V0_l^e(X) = V_{l_m}^e(X_p^{e*})\left(\frac{X}{X_p^{e*}}\right)^{\beta_1} \tag{4.45}$$

where X_p^{e*} and Q_l^{e*} are numerically obtained by simultaneous solving equations:

$$V_{l_m}^{e}\left(X_p^{e*},Q_l^{e*}\right)=V_{0f}^{e}\left(X_p^{e*},Q_l^{e*}\right) \tag{4.46}$$

and

$$\left.\frac{\partial V_{l_m}^{e}\left(X,Q_l^{e*}\right)}{\partial Q_l^{e*}}\right|_{X=X_p^{e*}}=0 \tag{4.47}$$

The optimal market price for investing is given by:

$$P_l^{e*}=X_p^{e*}\left(1-\eta Q_l^{e*}\right) \tag{4.48}$$

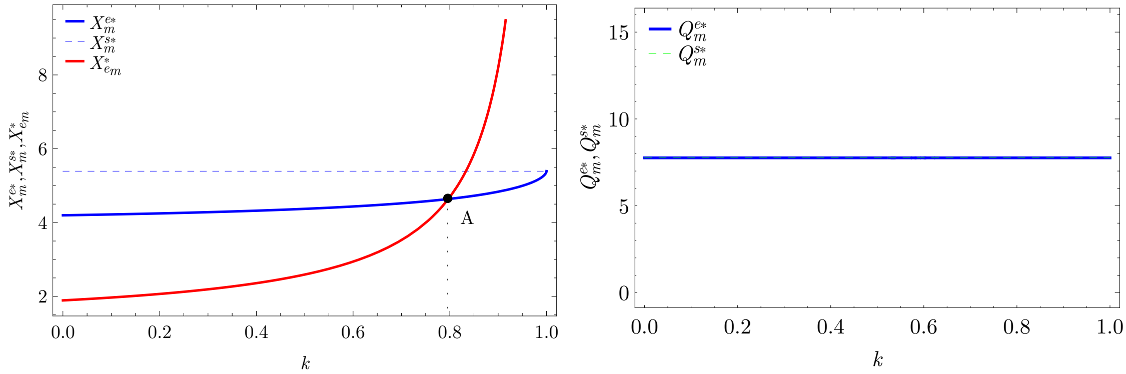
Similar to the last case, the leader also has to consider the two situations mentioned in Proposition 10. For values of $X_{e_{l_m}}^{*}<X_f^{e*}$, the leader knows that its monopolist rents will not end because of the follower's investment, but because the government will expropriate him first. Under this circumstances the correction to the value of the active leader, given by $V_{l_m}^{e}(X)$ (4.38), takes into account, similar to the case of the risky monopolist, that the cash-flows from operating the business, in a monopoly, will eventually be worth nothing, as X approaches $X_{e_{l_m}}^{*}$. As mention before, the value that the government attributes to the leader its the value of the leader as a monopolist corrected for the eventual entrance of the follower in the market. So, its value at expropriation, the first moment X hits $X_{e_{l_m}}^{*}$, given by $V_{e_{l_m}}$ (4.37), is the fraction of the corrected value of the monopolistic rents.

For the second case, $X_{e_{l_m}}^{*}>X_f^{e*}$, the leader knows that after investment, the follower will be the next to enter the market, the first moment X hits X_f^{e*}, and only after the follower enters the market, the government might expropriate him. The leader not only needs to incorporate in his value the value of the option held by the follower but also has to incorporate the option of the government to expropriate him. Back to front, the value of the leader at the moment of expropriation, given by $V_{e_{l_d}}$ (4.42), its the fraction of the value of the leader's rents at that moment, that the government compensates him for the expropriation. In this case, the rents while in a duopoly since the follower has already invested. Before expropriation, and after the follower invests, the value of the leader, given by $V_{l_d}^{e}(X)$ (4.43), its the present value of the cash-flows, from being in a duopoly, with expiration at the moment the government expropriates plus the value of the compensation from being expropriated. After the the leader invests, and before the follower invests, the value, given by $V_{l_m}^{e}(X)$ (4.44), its the value the of cash-flows from being a monopolist in the market corrected by the follower's option to invest, with the governments option to expropriate embedded in it.

4.3 Numerical Results

The base-case parameter values are: $r=0.06$, $\mu=0.02$, $\sigma=0.25$, $c_v=0.4$, $c_g=0.6$, $\delta=20$, $\eta=0.05$, $\rho=0$, $\tau=0.15$ and $k=0.85$. For the competitive case $k=0.90$ is used in order to allow for both firms to enter the market before expropriation could happen, otherwise, for lower values of k, the leader would be expropriated before the follower enters the market.

4.3.1 Monopoly



(a) X_m^{e*} and X_{em}^* (b) Q_m^{e*}

Figure 4.1: Sensitivity of X_m^{e*}, X_{em}^* and Q_m^{e*} to k

One important incite comes from the results in Figure 4.1. In Figure 4.1a we can see that for values of $k < 1$, the optimal time to invest for the risky monopolist, X_m^{e*}, is lower than the optimal to invest for the safe monopolist, X_m^{s*}. This means that the risky monopolist will invest sooner than the safe monopolist. Because the monopolist knows that it will be expropriated once a threshold is reached, it will try to invest sooner than normal so it can be in the market for longer before being expropriated to capture more cash-flows. As the value of k increases, the trigger of the risky monopolist converges to the safe case, until the point $k = 1$ where they become the same. At the point $k = 1$ the risky case turns into the safe case and the results in the subsection 3.3.1 of the Chapter 3 apply.

Point "A" in Figure 4.1a, denotes the level of k for which the trigger for expropriation X_{em}^* equals the investment trigger for the risky monopolist X_m^{e*}. This point is the last value k where the risky monopolist will not invest since it would be immediately expropriated. For values of $X_m^{e*} \leq X_{em}^*$ the risky monopolist will not invest, otherwise it will.

Figure 4.1b has an interesting result, as it shows that the optimal installed capacity that a firm investing in a country where there is a risk of expropriation is the same as if it was investing in a safe country. In other words, political risk seems not to influence the level of investment of a monopolist, but only its timing. So we have that the monopolist will try to reduce the value of the business, to reduce its "appeal" to the government, not by reducing its quantity but by investing sooner at a lower market price.

Looking now at Figure 4.2, where the evolution of the price in this risky market where only a monopolist exists, P_m^e, is represented, we can see a better picture of what is going on when the risky monopolist invests until the government expropriates. At the moment the risky monopolist invests, the initial price in the market is equal to the optimal trigger price to invest for the risky monopolist, P_m^{e*}. As time goes by (as X gets bigger), the price in the market rises until it hits a price ceiling for the risky monopolist where it will be removed from the market. That price ceiling is the trigger price for expropriation for the government, P_{em}^*.

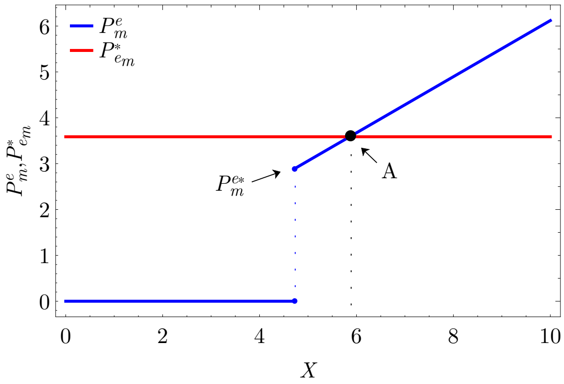
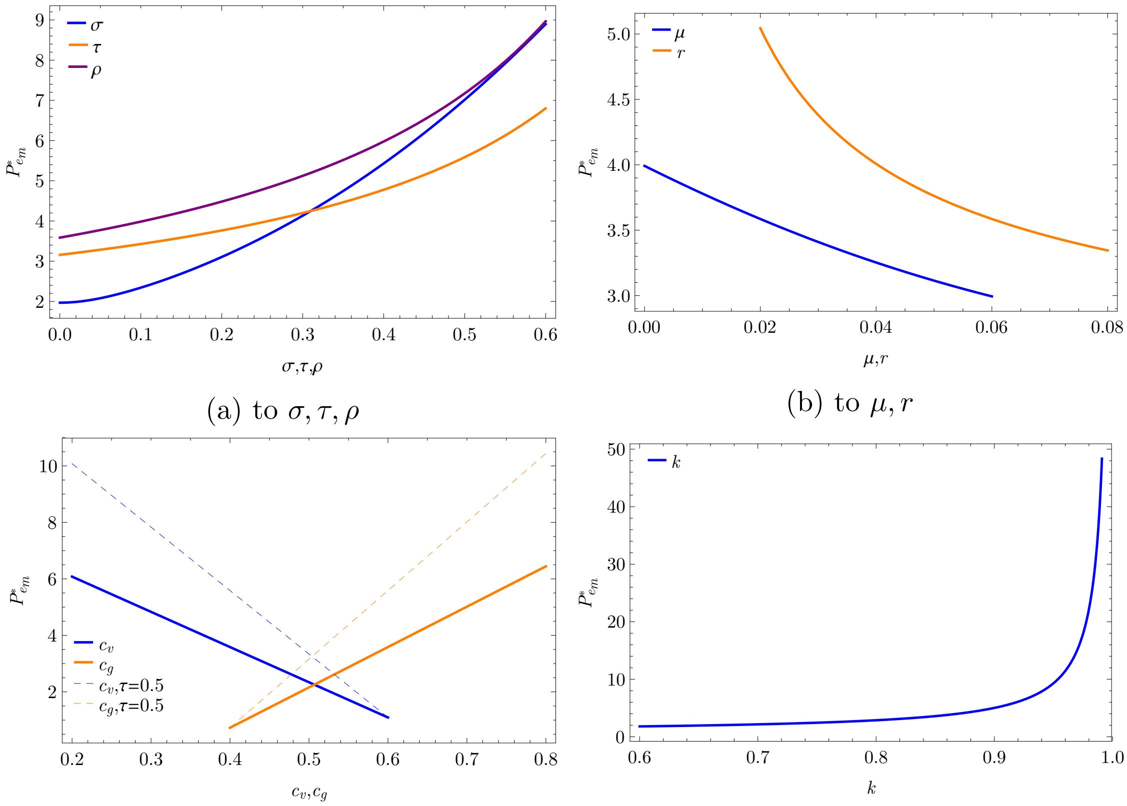


Figure 4.2: Market price in a risky monopolistic setting, P_m^e, for values of X, accounting for the expropriation price, $P_{e_m}^*$



(a) to σ, τ, ρ

(b) to μ, r

(c) to c_v, c_g

(d) to k

Figure 4.3: Sensitivity of $P_{e_m}^*$ to its variables, moved independently

Let us look at the optimal price, $P_{e_m}^*$, at which the government will expropriate the risky monopolist. Although we could look at the optimal time, $X_{e_m}^*$, instead of the optimal price, we have to bear in mind that the government, in the end, is looking for the price that makes it optimal to give up the tax collected from the monopolist to gain control of the cash-flows from operating the business. Also note that the optimal time is linked to the optimal price, since as X grows, keeping the market quantity constant, the price in the market also grows, and thus a mutual conclusion can be achieved.

From Figure 4.3a we can see that as the volatility parameter, σ, increases, the price demanded for expropriation also increases. The explanation for this is similar to what was already mentioned before. As volatility increases, agents want a bigger buffer to compensate for the larger swings. Additionally, we can see that as the amount of taxes collect increase, τ and ρ, governments will tend to wait for higher prices in the market. Since the amount of taxes collected is larger, only higher prices will compensate for the loss of those taxes from operating the business, and thus, we can say that for higher amounts of taxes collected, governments will tend to wait longer to expropriate. This result is similar to Restrepo Ochoa et al., 2015, the difference being that in their paper they consider the optimal time and not the price in the market.

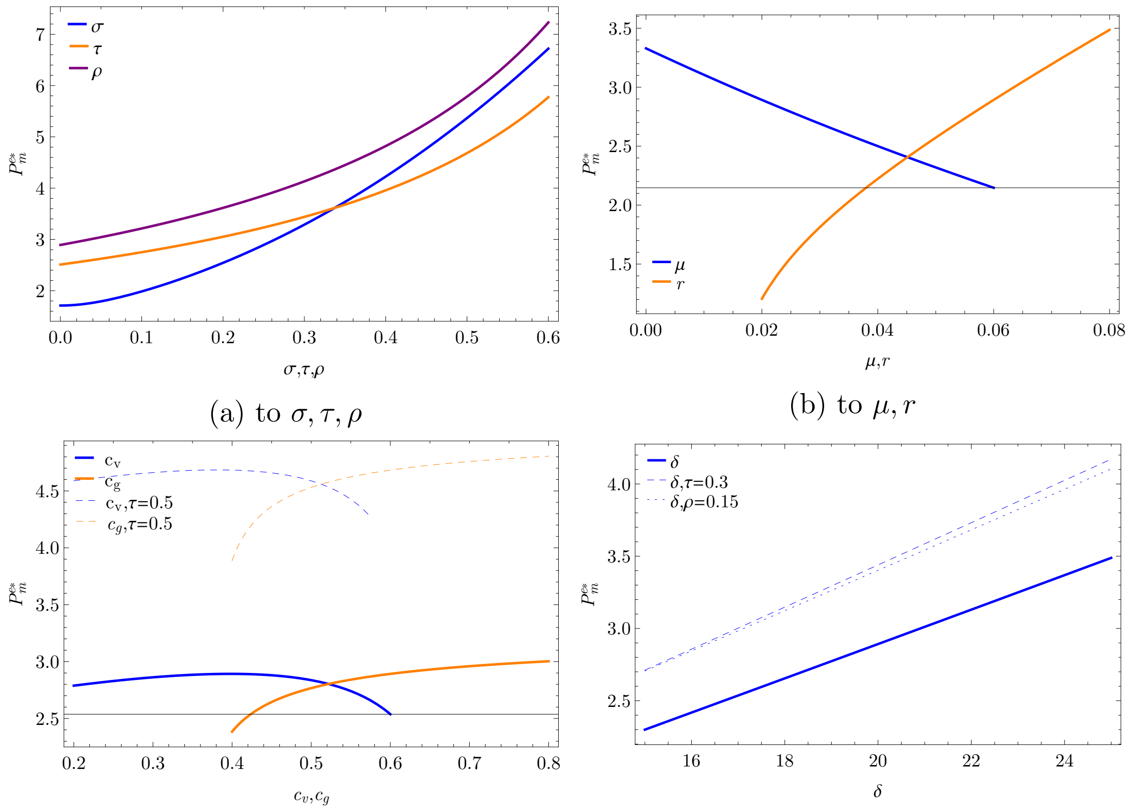
With respect to μ and r, we see from Figure 4.3b, that as μ increases the government will tend to expropriate sooner at a lower price, since higher growth in the market symbolizes potential for higher cash-flows that eventually compensate the higher costs from operating the business that the government has in comparison with the private sector. The parameter r, whoever, is the opposite of what is expected when we are talking about a private firm. As r increases, the optimal price to expropriate the risky monopolist decreases. One explanation can be that the government gives less value to lower present values of the tax collected than to lower present values from operating the business, and thus will expropriate sooner at a lower price, since for higher values of r the present value of the operating costs for the government, c_g, decrease.

Figure 4.3c results show, as expected, that as the production costs of the government, c_g, approach (diverge) the production costs of the private firm, c_v, it will tend to expropriate sooner (later) at a lower (higher) price. This comes from the simple explanation that as the government's efficiency of running the private firm approaches the efficiency of the private sector, the government will have the incentive to run the business himself and gain from the higher cash-flows. If, whoever, the government is so inefficient that it is better off collecting taxes, it will simply stay put and wait for higher prices that could compensate for the lack of efficiency.

In the case of the risky monopolist, because the intuition that we get by moving each individual parameter of the optimal price or optimal time to invest, is identical, to conserve space only the optimal price is shown (Figure 4.4). Beware that the intuition walks the same way for both cases.

Figure 4.4 shows the sensitivity of the optimal price to invest of the risky monopolist to its variables. The results are similar to those of the safe case with similar explanations. As σ, τ, ρ (Figure 4.4a), r (Figure 4.4b) and δ (Figure 4.4d) increase, the price demanded to enter the market also increases. For increasing values of μ (Figure 4.4b) the price decreases. The difference comes when we move the parameter for the costs of production of the private firm, c_v, and of the government, c_g (Figure 4.4c). As the efficiency of production of the private firms, c_v, approaches the efficiency of the government c_g, the trigger to invest for the risky monopolist starts to flatten, tending to diminish, meaning that the risky monopolist will invest sooner at a lower price, bearing in mind that we still need to ensure that $X_m^{e*} < X_{e_m}^*$.

Figure 4.5 shows the values of compensation k, for different parameter values,



(a) to σ, τ, ρ

(b) to μ, r

(c) to c_v, c_g

(d) to δ

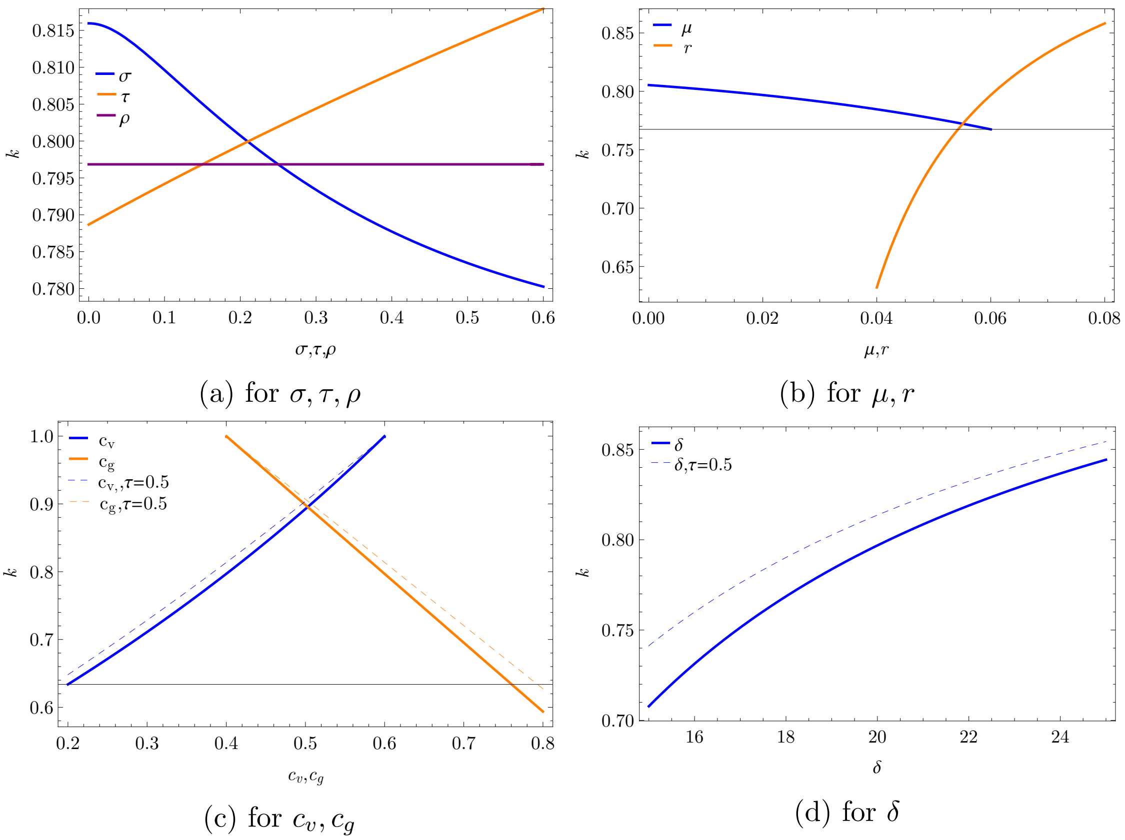
Figure 4.4: Sensitivity of P_m^{e*} to its variables, moved independently

that make the investment trigger of the risky monopolist, X_m^{e*}, be equal to the trigger to expropriate the risky monopolist, $X_{e_m}^*$. In other words, is the biggest value of k by which the government will still expropriate the monopolist immediately after investment. For higher values the monopolist will invest, for lower values it will not. In simple terms we have that for parameters where the trigger to invest X_m^{e*} decreases (increases) faster than $X_{e_m}^*$, the government will be required to pay less (more) for the monopolist to invest.

From Figure 4.5a it is possible to see that as volatility, σ, increases, the government does not have to compensate as much the risky monopolist. This result suggests that the monopolist's trigger for investment increases slower than the trigger for expropriation, for increasing values of σ.

The same cannot be said for the corporate tax, τ. As corporate tax increase, the amount of compensation needed also increases even thou increases in corporate tax tends to make the government expropriate later.

A different story can be seen if the parameter ρ, representing royalty's, is moved. It has no effect in the amount of compensation that the government must pay to ensure investment. The explanation for this result comes from the nature of royalty's. Because they directly affect the "real" sales price for the private firm, the response by firms is to look at the market price with the correction in mind, so it simply implies that firms will wait longer for the "real" price to archive the desirable goal, and the response of the expropriating government is also to wait longer since



(a) for σ, τ, ρ

(b) for μ, r

(c) for c_v, c_g

(d) for δ

Figure 4.5: Values of k, for different parameter values, that make $X_m^{e*} = X_{em}^*$

now, indirectly, it will have a bigger piece of the business. These two have the same weight in each other response.

In Figure 4.5b we can see that as μ increases the government does not have to compensate as much the risky monopolist in order for the monopolist to enter the market. With respect to r the story is the opposite. Since for higher values of r, the monopolist will tend to invest later but the government will tend to expropriate sooner, the impact on the amount of compensation tends to be more dramatic than for the other cases we have seen, and so we have that for increasing (decreasing) values of r the compensation needed tends to increase (decrease) rapidly.

Figure 4.5c shows that as the operating efficiency of the monopolist and government converge, the government must compensate the monopolist at a higher rate, otherwise the monopolist will not have the confidence that it would not be expropriated immediately since the government could be sufficiently efficient to run the business himself. By given a higher compensation the leader will feel more confident to invest since now the government will have a higher expropriation cost that offsets its hypothetical efficiency.

We also need to consider the amount of investment that the risky monopolist may need to do. As the investment commitment increases, the risky monopolist will ask for higher guarantees in order to commit to the investment. Figure 4.5d shows that as the cost per unit of installed capacity, δ, increases, the risky monopolist will ask for a higher compensation to offset the higher costs of investment. This result is expected since only the private firm faces investment costs, and so we have that δ does not affect the trigger for expropriation. Its possible to deduce that investment-intensive industry will be more prone to ask for bigger guarantees in order to invest since they will have a higher δ.

4.3.2 Competitive

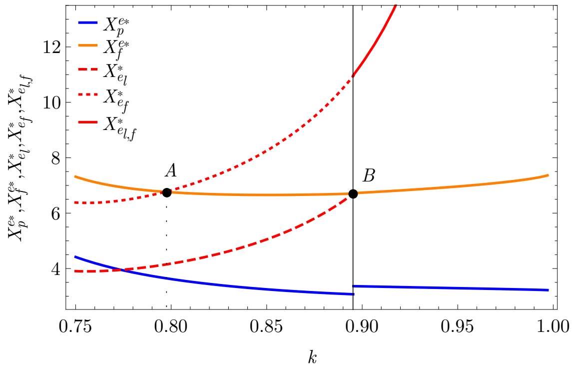


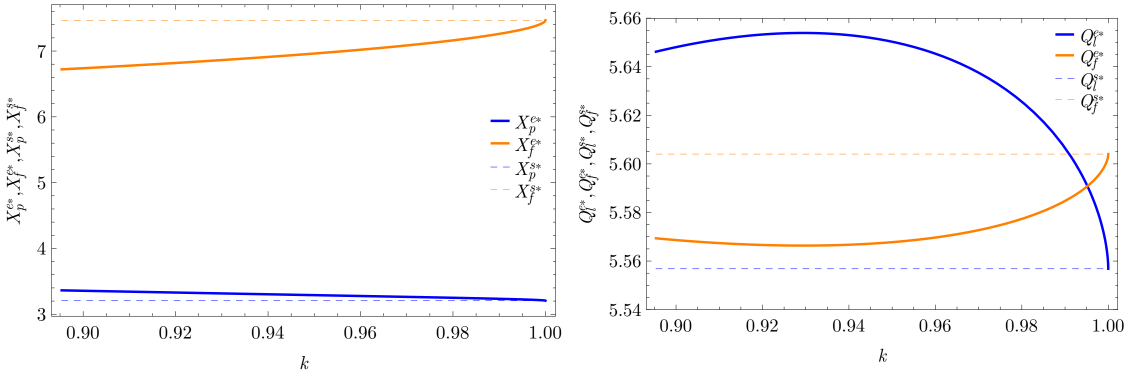
Figure 4.6: Sensitivity of X_p^{e*}, X_f^{e*}, $X_{e_{l_m}}^*$, $X_{e_f}^*$ and $X_{e_{l_d}}^*$ to k

From Figure 4.6 we can see that the first moment in which a market can be formed happens to values of k higher than point "A", where "A" is the first moment when $X_f^{e*} = X_{e_f}^*$. Note that point "A" in Figure 4.6 is the same point "A" in Figure 4.1a which means that the lower limit of compensation that makes a monopoly or a duopoly exist is the same. For values of k lower than this point the follower will not invest because the trigger for expropriation, $X_{e_f}^*$, is lower than the trigger for investment X_f^{e*}. For values of $A \leq k \leq B$, where "B" is the point where the trigger to expropriate the leader is equal to the trigger of the follower to invest, $X_f^{e*} = X_{e_{l_m}}^*$, the leader will be expropriated before the follower invests, which means that the follower, when it invests, will be competing with the government.

Because it is assumed that private firms will not invest in a market where there is an operating government enterprise, first we need to set that a competitive market, in a country where risk of expropriation exist, will only be formed if the follower enters the market before the leader is expropriated, otherwise the follower would be investing in a market where the government would be operating.

In Figure 4.6, the last point where the leader will be expropriated before the follower invests is denoted by point "B", where $X_f^{e*} = X_{e_{l_m}}^*$. So, for values of k higher than point "B" the follower will invest and a competitive setting populated by private firms will exist. By making this assumption, we are also assuming that for values of $A < k \leq B$, the only market that will be formed is a monopolistic one, since in this range the follower will not invest, otherwise it would be competing with the government.

Since the trigger to expropriate the leader and the follower, after both invest, is almost identical to the monopolist case, the only difference being the quantity in the market, the movement to its variables is identical to monopolistic case (Figure 4.3), albeit at a higher trigger. To conserve space see the explanation in the monopolistic case.



(a) X_p^{e*} and X_f^{e*} (b) Q_l^{e*} and Q_f^{e*}

Figure 4.7: Sensitivity of X_p^{e*}, X_f^{e*}, Q_l^{e*} and Q_f^{e*} to k

Given the base-case, if we move the variable k with respect to the investment trigger for the risky leader, X_p^{e*}, and risky follower X_f^{e*}, and comparing with the safe case (Figure 4.7) we can conclude that the risky leader will invest sooner and the follower will invest later than the respective safe case. Because the leader knows that it will not be expropriated before the follower invests, it will tend to invest later, and conversely, the follower will invest sooner because it knows that it will be expropriated after it invests.

From Figure 4.7b we can see that the leader will invest in more installed capacity, Q_l^{e*}, than in the safe case, Q_l^{s*}, and the risky follower it will invest less in installed capacity, Q_f^{e*}, when compared to its safe case, Q_f^{s*}. This means that, given that the government only expropriates the private firms after the follower enters the market, the leader will tend to take a bigger share of the market when compared to the safe case.

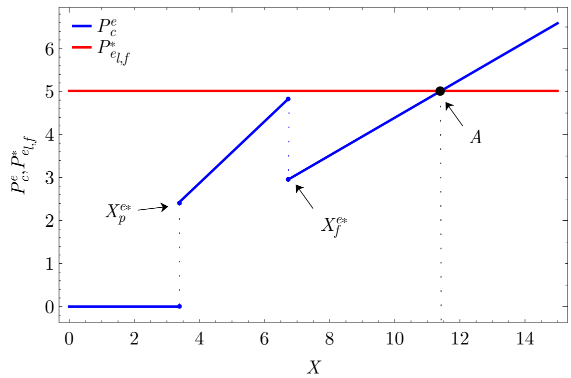
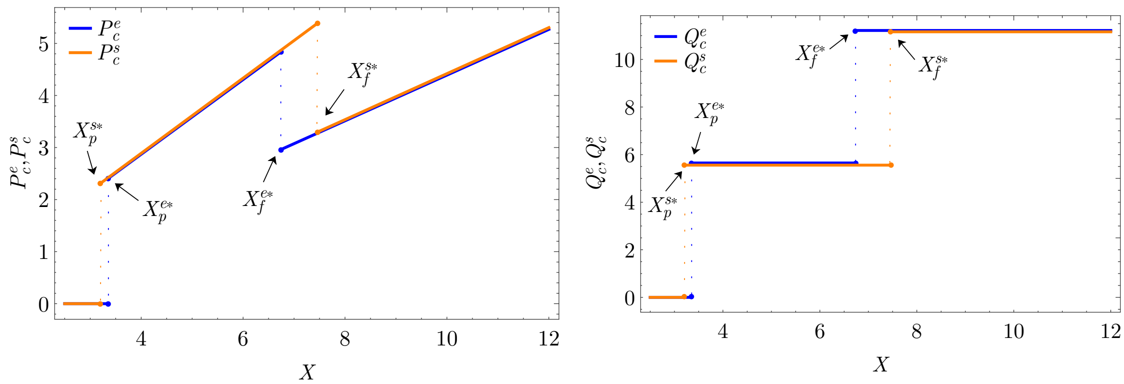


Figure 4.8: Market price in a risky competitive setting, P_c^e, for values of X, accounting for the expropriation price, $P^*_{e_{l,f}}$

Since, as we saw in Figure 4.6, the last point where the leader would be expropriated was point "B", at around $k = 89.53\%$, by choosing $k = 90\%$ for our base case, we ensure, barely, that the follower will invest before the leader is expropriated. The market picture is given by Figure 4.8. In it we can see that the market price ceiling, that once hit will make the government expropriate, $P^*_{e_{l,f}}$, is high enough for the leader to invest as monopolist, X_p^{e*}, and for the follower to invest, at X_f^{e*}, and turn the market in to a duopoly. Point "A" marks the moment when the price for expropriation equals the market price in the risky competitive setting, $P_{e_{l,f}}^{*} = P_c^e$. At this point, the government will expropriate both firms and becomes the monopolist in the market.



(a) P_c^s and P_c^e

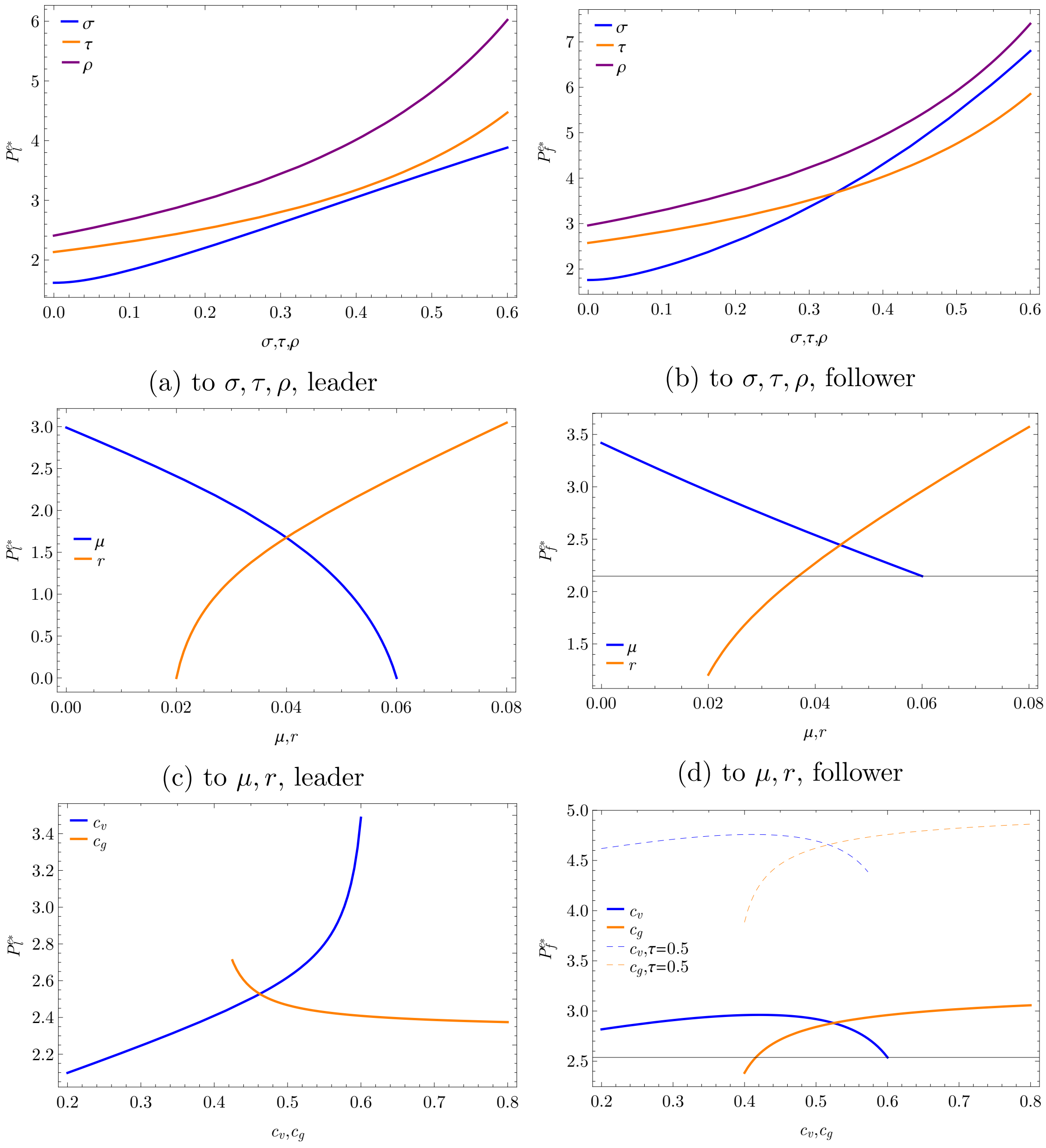
(b) Q_c^s and P_c^e

Figure 4.9: Comparison of the safe and risky market prices, P_c^s and P_c^e, and quantity's, Q_c^s and P_c^e

Figure 4.9 shows the full picture of the differences between the safe and risky case. From Figure 4.9a we can see that the risky leader will invest later at a higher market price, and the risky follower will invest sooner at a lower market price than the respective safe cases. We also can take away from 4.9b that the risky market will be deprived of the product/service for longer initially, but because the risky follower invests sooner than in the safe case, the market supply will increase sooner than in the safe case. We also can take away that for values where both scenarios are in the same range of X, the market price and quantity of both scenarios do not seem to be much different.

Figure 4.10 shows the sensitivity of P_l^{e*} and P_f^{e*} to their variables. If we look at the Figures for P_f^{e*} (Figure 4.10b, 4.10d and 4.10f) we can see that movement in relation to the variables is virtually the same as for the risky monopolist (Figure 4.2) with similar intuition. For the risky leader, the intuition is similar to the safe leader's case, but we have to take into consideration that the risky leader has to consider the costs of production of the government (Figure 4.10e). As the costs of production of the government, c_g, get close to the costs of production for the private firm, c_v, the risky leader will tend to invest at higher market prices. The variability to the investment costs are not shown, but the intuition is straightforward, higher investments costs make the leader or the follower invest later at a higher market price. Because the amount of compensation that is being considered in the base-case, $k = 90\%$, is close to the limit amount for which a competitive market will emerge, $k = 89,53\%$, the results in Figure 4.10 are only demonstrative of the variation of the price triggers, since a small movement in some variables can make $k = 90\%$ insufficient to ensure that the follower invest before the leader is expropriated.

Similar to the case of the risky monopolist, we want to know, for different parameters, the last value of k by which a competitive market will be formed. In this



(a) to σ, τ, ρ, leader

(b) to σ, τ, ρ, follower

(c) to μ, r, leader

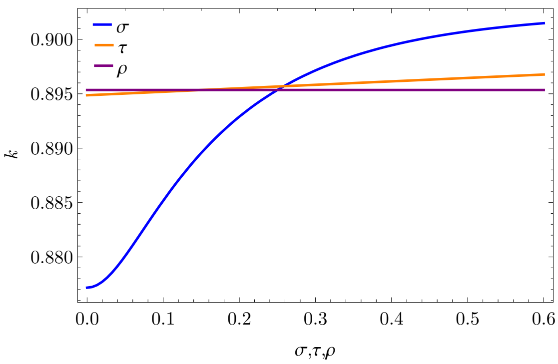
(d) to μ, r, follower

(e) to c_v, c_g, leader

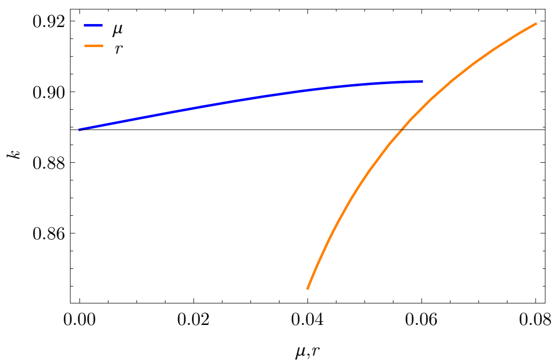
(f) to c_v, c_g, follower

Figure 4.10: Sensitivity of P_l^{e*} and P_f^{e*} to its variables, moved independently

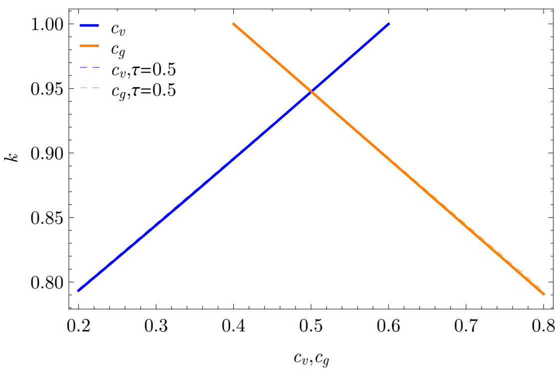
case values that make $X_f^{e*} = X_{e_{l_m}}^*$. The results are given by Figure 4.11. In it, we can see that the only difference in movement from the monopolistic case, Figure 4.5, come from σ and μ. The movement of the others are similar and with similar intuition, albeit with higher values of k. Different from the risky monopolist case, the investment trigger of the follower increases faster for increasing values of μ, σ than the trigger to expropriate the leader while monopolist, which means that the government has to increase its compensation to offset the difference.



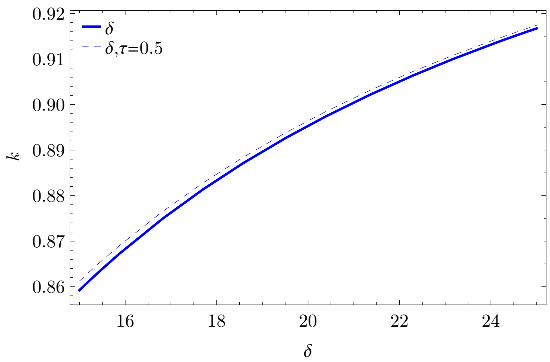
(a) for σ, τ, ρ



(b) for μ, r



(c) for c_v, c_g



(d) for δ

Figure 4.11: Values of k, for different parameter values, that make $X_f^{e*} = X_{e_{l_m}}^*$

Chapter 5

Choice of Investment Place

This Chapter explores the possibility for firms to produce in their home country to then export their product to the foreign market, instead of producing in the country where they want to sell there product. This way firms are less exposed to events in the foreign country, like expropriation, simply because there is no means of production to be expropriated. Of course, there are other ways to diminish the exposure like licensing or joint ventures, but for simplification only exporting is studied.

Since, in theory, firms can mismatch were they invest, home or foreign, the superscript $n = \{i, h\}$ is used to encapsulate all the possibilities, where h indicates investing in the home country, and $i = \{s, e\}$, as before, represents investing in a safe/risky foreign country. In practice, because firms are symmetric, the best choice for one is also the best choice for the other.

Because firms are exporting products to a foreign country, additional costs have to be considered. In addition to the cost already mentioned, transportation cost, denoted by γ, and a tariff tax, denoted by θ, are also added. Also to account for the differences in the investment costs to produce and sell in the foreign country or produce in the home country and then sell in the foreign country we note $\delta_f = \delta$ and δ_h as the investment costs per unit of Q for the foreign and home case, respectively.

We still need to consider that a firm exporting production to then sell it in the foreign country needs to act in conformity with rules that regulate transactions between firms under common ownership and that the rule of dealing at arm's length applies, since now we are considering that a firm, in practice, will have revenues in the home country, in the form of exports, and revenues in the foreign country, in the form of sales to the final consumer. The transfer price considered, denoted by P_h:

$$P_h = (c_h + \gamma)(1 + \phi) \tag{5.1}$$

is obtained by applying an exogenous markup percent value, denoted by ϕ, to the costs of producing in the home country, denoted by c_h, and the transportation costs. This markup value is thought to be the normal markup applied in the industry for transactions between firms.

5.1 Producing in the the Home country

Proposition 12. *Investment option value for the Monopolist producing in the home country:*

The cash flows accruing to the monopolist producing in the home country, at time t, are given by:

$$\pi_m^h(t) = Q_m^h(t)(P_h - c_h - \gamma)(1-\tau_h) \\ + Q_m^h(t)\left(X(t)\left(1-\eta Q_m^h(t)\right)(1-\rho) - P_h(1+\theta)\right)(1-\tau_f) \quad (5.2)$$

and the expected value of the cash-flows at the moment of investment is given by:

$$\Pi_m^h\left(X, Q_m^h\right) = E\left[\int_{t=0}^{\infty} \pi_m^h(t)\, e^{-rt} dt\right] \\ = Q_m^h \frac{P_h - c_h - \gamma}{r}(1-\tau_h) + Q_m^h\left(\frac{X(1-\eta Q_m^h)}{r-\mu}(1-\rho) - \frac{P_h(1+\theta)}{r}\right)(1-\tau_f) \quad (5.3)$$

The value of the monopolist producing in the home country, after investing, $X \geq X_m^{h*}$, is given by:

$$V_m^h(X) = \Pi_m^h\left(X, Q_m^{h*}\right) - \delta_h Q_m^{h*} \quad (5.4)$$

and before investing, as it still holds the option to invest, $X < X_m^{h*}$, is:

$$V0_m^h(X) = \left(\Pi_m^h\left(X_m^{h*}, Q_m^{h*}\right) - \delta_h Q_m^{h*}\right)\left(\frac{X}{X_m^{h*}}\right)^{\beta_1} \quad (5.5)$$

where

$$X_m^{h*} = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)\left(c_h + r\delta_h + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right)P_h\right)}{r(1-\rho)(1-\tau_f)} \quad (5.6)$$

is the optimal time to invest, and

$$Q_m^{h*} = \frac{1}{(\beta_1+1)\eta} \quad (5.7)$$

is the optimal capacity at the moment of investment.

The optimal market price for investment is given by:

$$P_m^{h*} = X_m^{h*}\left(1-\eta Q_m^{h*}\right) \quad (5.8)$$

The present value of the cash-flows accruing to the monopolist, producing in its home country, are treated as the present value of a perpetuity since it is assumed that the government, because the monopolist is producing in his home country, does not have the possibility to expropriate the business. The value of the cash-flows, given by $\Pi_m^h(X, Q_m^h)$ (5.3), account for the extra costs in comparison to the previous cases, namely the addition of transportation costs, γ, the corporate tax that must be paid in the home country, τ_h, and the tariff tax θ. In the home country, the corporate tax is applied to the home profits that are calculated as being the transfer price deduced from the production and transportation costs multiplied by the amount produced. In the foreign country the corporate tax, noted by τ_f, is applied on the foreign profits that are consider to be the percent of the reselling price (market price) left after the royalty fee is applied, deduced of the costs of buying the product from the home country, that may be subject to a tariff tax.

The value of the monopolist at the moment of investing, to produce in the home country, the first time X hits X_m^{h*}, given by $V_m^h(X)$ (5.4), its the value of the cash flows from operating the business from the home country (equation 5.3) minus the investment costs in installed capacity $\delta_h Q_m^{h*}$. Before investing, for values of $X < X_m^{h*}$, the value of the monopolist producing at home is the value of the option to invest that it holds, denoted by $V0_m^h(X)$ (5.5).

Proposition 13. *Investment option value for the Follower producing in the home country:*

The cash flows accruing to the follower producing in the home country, at time t, are given by:

$$\pi_f^h(t) = Q_f^h(t)(P_h - c_v - \gamma)(1-\tau_h) \\ + Q_f^h(t)\left(X(t)\left(1-\eta\left(Q_l^n(t)+Q_f^h(t)\right)\right)(1-\rho) - P_h(1+\theta)\right)(1-\tau_f) \quad (5.9)$$

and the expected value of the cash-flows at the moment of investment is given by:

$$\Pi_f^h\left(X, Q_l^n, Q_f^h\right) = E\left[\int_{t=0}^{\infty} \pi_f^h(t)\, e^{-rt} dt\right]$$
$$= Q_f^h \frac{P_h - c_h - \gamma}{r}(1-\tau_h) + Q_f^h\left(\frac{X\left(1-\eta(Q_l^n + Q_f^h)\right)}{r-\mu}(1-\rho) - \frac{P_h(1+\theta)}{r}\right)(1-\tau_f) \quad (5.10)$$

The value of the follower producing in the home country after investing, $X \geq X_f^h(Q_l^n)$, is given by:

$$V_f^h(X, Q_l^n) = \Pi_f^h\left(X, Q_l^n, Q_f^{h*}(Q_l^i)\right) - \delta_h Q_f^{h*}(Q_l^n) \quad (5.11)$$

and before investing, as it still holds the option to invest, $X < X_f^h(Q_l^n)$, is:

$$V0_f^h(X, Q_l^n) = \left(\Pi_f^h\left(X_f^{h*}(Q_l^n), Q_l^n, Q_f^{h*}(Q_l^n),\right) - \delta_h Q_f^{h*}(Q_l^n)\right)\left(\frac{X}{X_f^{h*}(Q_l^n)}\right)^{\beta_1} \quad (5.12)$$

where

$$X_f^{h*}(Q_l^n) = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)\left(c_h + r\delta_h + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right)P_h\right)}{r(1-\rho)(1-\tau_f)(1-Q_l^n)} \quad (5.13)$$

is the optimal time to invest, and

$$Q_f^{h*}\left(Q_l^n\right) = \frac{1-\eta Q_l^n}{(\beta_1+1)\eta} \tag{5.14}$$

is the optimal capacity at the moment of investment.

The optimal market price for investing is given by:

$$P_f^{h*}\left(Q_l^n\right) = X_f^{h*}\left(Q_l^n\right)\left(1-\eta\left(Q_l^n+Q_f^{h*}\left(Q_l^n\right)\right)\right) \tag{5.15}$$

The value of the follower, producing at home, after investing, given by $V_f^h(X, Q_l^m)$ (5.11), is the present value of the cash-flows that the follower gets from producing at home and selling in the foreign country, noted by equation (5.10), minus the initial investment cost $\delta_h Q_f^{h*}(Q_l^n)$. Before investing, for values of $X < X_f^{h*}(Q_l^n)$, the follower's value, given by $V_{0f}^{\;h}(X, Q_l^n)$ (5.12), its equal to the investment option value that it holds to produce in the home country.

Proposition 14. *Investment option value for the Leader producing in the home country:*

The cash flows accruing to the leader producing in the home country, before the follower invests, at time t, are given by:

$$\begin{aligned}\pi_{l_m}^h(t) &= Q_l^h(t)\left(P_h-c_h-\gamma\right)\left(1-\tau_h\right)\\ &\quad + Q_l^h(t)\left(X(t)\left(1-\eta Q_l^h(t)\right)(1-\rho)-P_h(1+\theta)\right)\left(1-\tau_f\right)\end{aligned} \tag{5.16}$$

and after the follower invests are given by:

$$\begin{aligned}\pi_{l_d}^h(t) &= Q_l^h(t)\left(P_h-c_h-\gamma\right)\left(1-\tau_h\right)\\ &\quad + Q_l^h(t)\left(X(t)\left(1-\eta\left(Q_l^h(t)+Q_f^n(t)\right)\right)(1-\rho)-P_h(1+\theta)\right)\left(1-\tau_f\right)\end{aligned} \tag{5.17}$$

The expected value of the cash-flows at the moment the leader producing in the home country invests, and before the follower invests, is given by:

$$\begin{aligned}\Pi_{l_m}^h\left(X, Q_l^i\right) &= \mathrm{E}\left[\int_{t=0}^{\infty}\pi_{l_m}^h(t)\,e^{-rt}dt\right] + \mathrm{E}\left[\int_{T_f^{i*}}^{\infty}\left(\pi_{l_d}^h(t)-\pi_{l_m}^h(t)\right)e^{-rt}dt\right]\\ &= Q_l^h\frac{P_h-c_h-\gamma}{r}\left(1-\tau_h\right)+Q_l^h\left(\left(\frac{X(1-\eta Q_l^h)}{r-\mu}\right.\right.\\ &\quad \left.\left.-\left(\frac{X}{X_f^{n*}(Q_l^h)}\right)^{\beta_1}\frac{X_f^{n*}(Q_l^h)\eta Q_f^{n*}(Q_l^h)}{r-\mu}\right)(1-\rho)-\frac{P_h(1-\theta)}{r}\right)\left(1-\tau_f\right)\end{aligned} \tag{5.18}$$

and after the follower invests, given by:

$$\Pi_{l_d}^{h}\left(X,Q_l^h\right)=\mathrm{E}\left[\int\limits_{t=T_f^{i*}}^{\infty}\pi_{l_d}^{h}(t)\,e^{-rt}dt\right]$$
$$=Q_l^h\frac{P_h-c_h-\gamma}{r}(1-\tau_h)+Q_l^h\left(\frac{X\left(1-\eta(Q_l^h+Q_f^n)\right)}{r-\mu}(1-\rho)-\frac{P_h(1-\theta)}{r}\right)(1-\tau_f) \tag{5.19}$$

The value of the leader producing in the home country, after investing, $X_f^{n*}\left(Q_l^h\right)>X\geq X_p^{h*}$*, is given by:*

$$V_{l_m}^{h}(X)=\Pi_{l_m}^{h}\left(X,Q_l^{h*}\right)-\delta_hQ_l^{h*} \tag{5.20}$$

and before investing, as it still holds the option, $X<X_p^{h*}<X_f^{n*}\left(Q_l^h\right)$*, is:*

$$V0_l^h(X)=\left(\Pi_{l_m}^{h}\left(X_l^{h*},Q_l^{h*}\right)-\delta_hQ_l^{h*}\right)\left(\frac{X}{X_l^{h*}}\right)^{\beta_1} \tag{5.21}$$

where X_p^{h*}*, the leader's optimal time to invest, and* Q_l^{h*}*, the optimal capacity at the moment of investment, are obtained by simultaneously solving equations:*

$$V_{l_m}^{h}\left(X_p^{h*},Q_l^{h*}\right)=V0_f^n\left(X_p^{h*},Q_l^{h*}\right) \tag{5.22}$$

and

$$\left.\frac{\partial V_{l_m}^{s}\left(X,Q_l^{h*}\right)}{\partial Q_l^{h*}}\right|_{X=X_p^{h*}}=0 \tag{5.23}$$

The optimal market price for investing is given by:

$$P_l^{h*}=X_l^{h*}\left(1-\eta Q_l^{h*}\right) \tag{5.24}$$

The leader's case in this Chapter is similar to the leader's case in the safe environment, the only difference being the extra costs that it has to endure. Before the follower invests, $X<X_f^{n*}$, the value of the leader, given by $V_{l_m}^{h}(X)$ (5.20), needs to take into account the drop in value of the cash flows that will take place once the follower invests. After the follower invest, $X\geq X_f^{n*}$, the value of the leader is simply its value from being in a duopoly.

5.2 Rules for choosing

To decide what strategy is optimal for firms, to whether produce in there home country or produce in the foreign country, a rule of investment is in order. Firms, at the beginning of the process, already known that they have the option to produce in their home country or in the foreign country. The best optimal strategy is to choose the option with the most value the first time that they can observe which option is more valuable since these two options are mutually exclusive, firms can only decide to produce in one of two countries. Also, we have to take into consideration that firms need time to plan the implementation of their investments, and by choosing the optimal strategy the first time that they can observe which plan is better gives them more time to plan the investment.

If the foreign country is considered to be safe, in other words, the government does not intend to expropriate, the optimal strategy is to choose the option with the higher value between the two:

$$\max\left[V0^s_n, V0^h_n\right] \tag{5.25}$$

where the subscript $n = \{m, l(p), f\}$ represents the monopolist, the leader and the follower, respectively.

If, on the other hand, the foreign government acts opportunistically, first firms need to check if the trigger of expropriation is higher than their trigger for investment, otherwise the intrinsic value of the business would be zero, and their only choice is to produce in their home county:

$$\begin{cases} \max\left[V0^e_n, V0^h_n\right] & if \quad X^{e*}_n < X^*_{e_n} \\ V0^h_n & if \quad X^{e*}_n \geq X^*_{e_n} \end{cases} \tag{5.26}$$

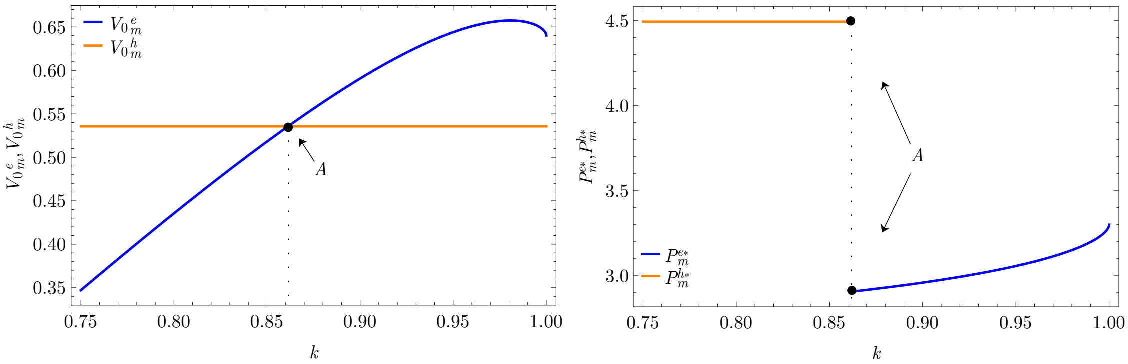
In the following section, we will only compare the case for producing at home with the risky foreign scenario, since that is what we are more interest in.

5.3 Numerical Results

The base-case parameter values are: $r = 0.06$, $\mu = 0.02$, $\sigma = 0.25$, $c_v = 0.4$, $c_h = 0.5$, $c_g = 0.6$, $\delta_f = 20$, $\delta_h = 25$, $\eta = 0.05$, $\rho = 0$, $\tau_f = 0.15$, $\tau_h = 0.20$, $\gamma = 0.1$, $\phi = 0.1$, $\theta = 0.15$ and $k = 0.85$. For the competitive case $k = 0.90$ is used.

5.3.1 Monopoly

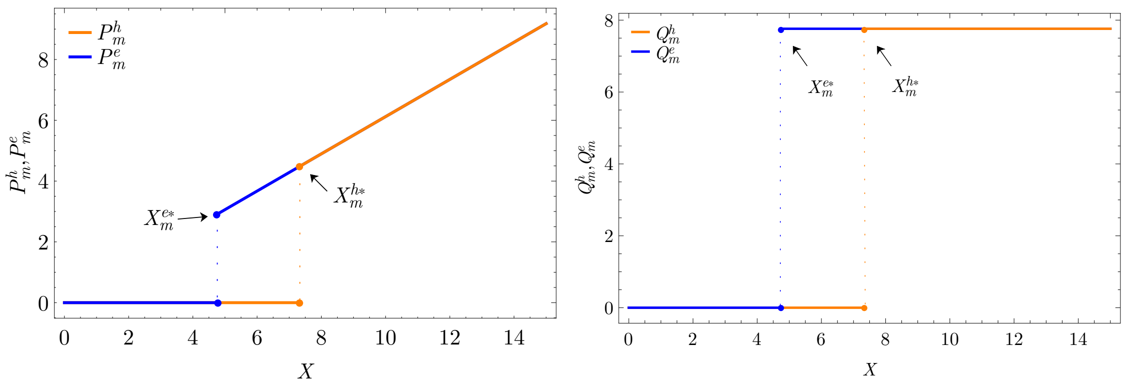
Given the base-case parameter, that assumes that the costs in the foreign country are lower than in the home country, the first thing that a firm needs to do in order to identify the optimal choice of place for production, is to check which option that it holds, to produce in the home country or in the foreign country, has the highest value. In Figure 5.1a, point "A" marks the last value of k for which the value of the option to produce in the home country, $V0^h_m$, equals the value of the option to produce in the risky foreign country, $V0^e_m$. At this point, the monopolist is indifferent between producing in the home or the foreign country. Note that this representation does not mean that the government can change its k at will in order to attract investment. The k parameter is set at the beginning of the process and does not change, the graph only shows all the possible results.



(a) $V0^e_m$ and $V0^h_m$ (b) P^{e*}_m and P^{h*}_m

Figure 5.1: Comparison of $V0^e_m$, for different values of k, with $V0^h_m$ and comparison of P^{e*}_m, for different values of k, with P^{h*}_m

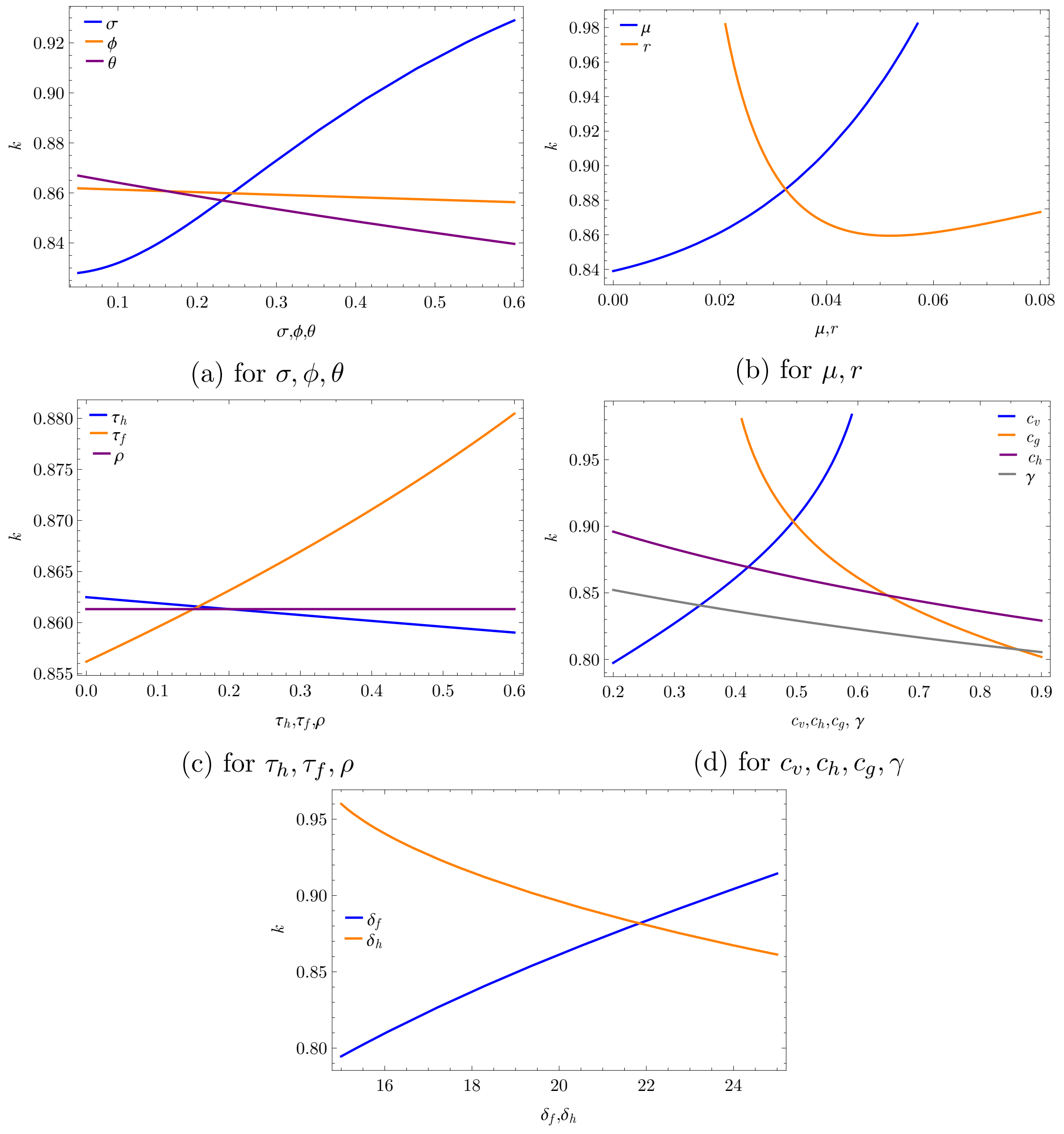
Because, for our base case of $k = 0.85$, the value of the option to produce in the home country has a higher value, so the monopolist will choose to produce in its home country to then export and sell in the foreign country, even though with higher and extra costs. From Figure 5.1b we can see that since that is the case, the initial market price will be higher, given by P^{h*}_m instead of P^{e*}_m, the trigger price to produce in the home country and the trigger price to produce in the foreign country, respectively. The higher initial price is to compensate for the extra costs that the monopolist has to endure for producing at home. Because the compensation for expropriation is not sufficiently high, the best strategy for the monopolist is to produce at home at a higher cost which is reflected in the price. Point "A" in Figure 5.1b shows the drop in the initial price (trigger price) that occurs when the monopolist decides to produce in the foreign country, when $V0^e_m > V0^h_m$.



(a) P^h_m and P^e_m (b) Q^h_m and Q^e_m

Figure 5.2: Comparison of the market price, in a monopolistic setting, from producing at home, P^h_m, or in the risky country, P^e_m, and the respective quantity's, Q^h_m and Q^e_m, for different values of X

Taking now a market perspective of our base-case, looking at Figure 5.2a, we can see that if the monopolist were to produce in the foreign country, it would invest sooner, X^{e*}_m, at a lower price, P^{e*}_m, when compared with producing in the home country, where it invests later, X_m^{h*}, and at a higher price, P_m^{h*}. Because the monopolist will produce in its home country and thus invests later, the market will be deprived of the potential product for longer (Figure 5.2b).



(a) for σ, ϕ, θ

(b) for μ, r

(c) for τ_h, τ_f, ρ

(d) for c_v, c_h, c_g, γ

(e) for δ_f, δ_h

Figure 5.3: Values of k, for different parameter values, that make $V_{0m}^h = V_{0m}^e$

Looking now at the amount of compensation that the government must pay to make $V_{0m}^e = V_{0m}^h$, for different parameter values (Figure 5.3), we can see from Figure 5.3a that as σ increases the amount of compensation must also increase. This result highlights the limitation that the government imposes on the value of the option to produce (invest) in the foreign country when compared to the option to produce at home country where there is no risk for expropriation. Because there is no risk for expropriation in the home country, as the volatility of the market increases the value of the option to produce in the home country is free to capture all the extra value that comes from the possibility for higher cash-flows, while the value of the option to produce in the foreign country is always limited by the existence of a government whiling to expropriate the firm. The markup factor, ϕ, does not seem to have much importance, given the base-case scenario. For higher values of ϕ the government will be able to pay slightly less compensation in order for the monopolist to invest in the foreign country, which can be explained by the double role that ϕ takes in the transfer price, that is both considered as revenue, in the home country, and as a cost, in the foreign country. In the case of the tariff tax θ, the story is different since it only affects negatively the value of the option to produce in the home country. Because θ reduces the value of the option to produce in the home country, the compensation that the government has to pay to ensure investment in the foreign country decreases.

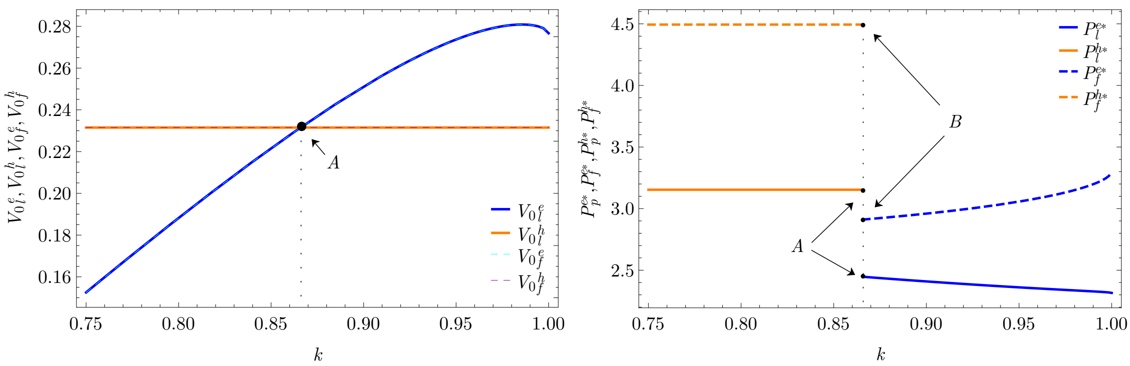
For the parameter μ (Figure 5.3b) the explanation for the results is similar to the parameter σ. Higher market growth increases the value of both options, but because the option to produce in the foreign country is limited by the existence of a government that will expropriate, the value of the option to produce at home grows faster. With respect to r we have that as it sufficiently small and decreases, it has the same effect as μ increasing, but for increasing and sufficiently high values o r the amount of compensation needed for the monopolist to invest in the foreign country decreases.

Higher corporate tax in the home country decrease the value of the option to produce in the home country and so the amount of compensation to make both options equal decreases. For vales of the tax ρ we have a similar story as we saw in previous chapters. Higher values of ρ only make the private firm wait for higher real prices in both situations (Figure 5.3c).

Turning now the production costs, c_h, transportation costs, γ, and investment costs, δ_h, in the home country we can see that as they increase, and thus as the value of the option to produce in the home country decreases, as expected, the amount of compensation needed decreases (Figure 5.3d and 5.3e). For the production costs in the foreign country, c_v, and the governments production costs, c_g, we need to take into consideration that both contribute to the value of the option to produce in the foreign country that the monopolist holds, and thus their variations can have a higher impact in the value of the option. If the government's efficiency is comparable to the private firm, it will need to pay more for expropriation. As its efficiency decreases, it will not be required to pay as much (Figure 5.3d). We also have that as the investment costs in the foreign country, δ_f, increase the amount of compensation also needs to increase to offset the loss in value of the option.

5.3.2 Competitive

For the competitive case we have to consider that if the compensation is not higher than $89,53\%$, and is optimal for the first firm to produce in the foreign country, the market will be composed of a single firm, since if a second one wanted to invest, it would be always competing with the government, since by the time it was optimal for it to invest the government would already expropriate the first firm and would be in the market. If it is optimal for the first firm to produce in the home country, since firms are symmetric, it will also be optimal for the second firm to produce in the home country.



(a) V_{0l}^e/V_{0f}^e with V_{0l}^h/V_{0f}^h

(b) P_l^{e*}/P_f^{e*} with P_l^{h*}/P_f^{h*}

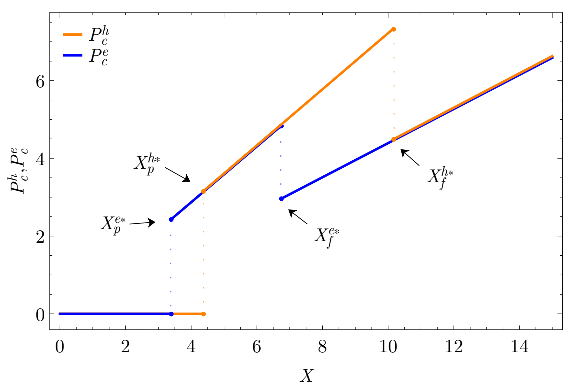
Figure 5.4: Comparison of V_{0l}^e/V_{0f}^e, for different values of k, with V_{0l}^h/V_{0f}^h, and comparison of P_l^{e*}/P_f^{e*}, for different values of k, with P_l^{h*}/P_f^{h*}

By assuming $k = 90\%$, from Figure 5.4a we can see that both firms will choose to produce in the foreign country, since the value of the respective options to produce in the foreign country, V_{0l}^e and V_{0f}^e, have a higher value than the options to produce at home, V_{0l}^h and V_{0f}^h. Point "A" in Figure 5.4a marks the moment when the value of the option to produce in the home country equals the value of the option to produce in the foreign country, for both leader and follower.

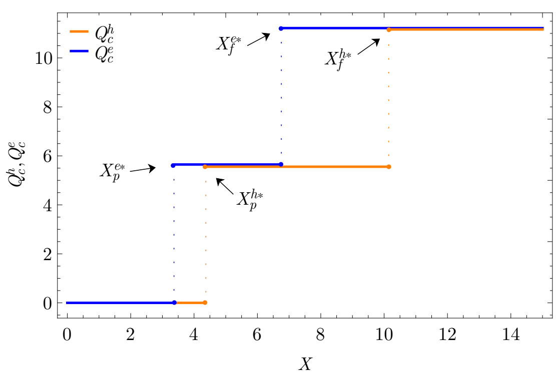
Figure 5.4b demonstrated the drop in price, similar to the monopolist case, that occurs when firms decide to produce in the foreign country instead of the home country. Point "A" shows the drop in price from the perspective of the leader and point "B" from the perspective of the follower. We can see that the drop in price tends to be larger for the case of the follower when compared to the leader.

For different parameter values, the intuition that we get about the amount of compensation that makes $V_{0l}^h = V_{0l}^e$, point "A" in Figure 5.4a, is the same as in the the monopolist case that we seen in Figure 5.3.

If we look at the market as a whole, we can see from Figure 5.5 that the market will be explored sooner and that the price will tend to be lower for longer if firms decide to invest in the country where the market is located. In other words, if a country wants to attract investment, so it gets lower prices and faster investments, its government needs to signal to private firms that the compensation that it gives for expropriation is sufficiently high that firms will be better of by choosing to invest in the foreign country.



(a) P_c^h and P_c^e



(b) Q_c^h and Q_c^e

Figure 5.5: Comparison of the market price, in a competitive setting, from producing at home, P_c^h, or in the risky country, P_c^e, and the respective quantity's, Q_c^h and Q_c^e

Chapter 6

Conclusions

The aim of this study was to model the effect of the amount of compensation that governments pay as compensation for expropriation in the investment decision of private firms, assuming that the government is an opportunistic agent that will expropriate private investments if it benefits from it.

In a monopolistic context the model shows that if the amount of compensation at expropriation is lower than the fair value of the monopolistic firm, the monopolist will tend to invest sooner but with the same installed capacity as if there was no expropriation risk. This means that the monopolist, knowing that the government will expropriate at a given market price, will try to invest sooner in a market where the demand for the product/service is yet to be optimal, so the market prices are low enough to create a larger margin between the price at investment and the expropriation price level. As the compensation approaches the fair value of the business, the investment strategy of the monopolist tends to its safe case.

For the competitive setting, assuming that firms don't want to compete with the government in the marketplace, since this government is thought to be non conformists it traditional fair competition arrangements between government-owned enterprises and privately owned enterprises, we have that the amount of compensation that the government needs to pay to ensure a competitive market is higher than for the monopolistic case. For the case that government does compensate enough private firms that a competitive market is formed, we have that the first private firm to invest (leader) will be tempted to invest later and with more quantity at a higher price, and the second firm (follower) will invest sooner and with lower capacity at a lower price, than the respective safe cases.

When we consider the possibility of private firms producing in their home country, where there is no expropriation risk but cost of production are higher, so they can avoid the risk of expropriation in the foreign country, we conclude that if firms decide to produce in their home country, the market price that they will demand to invest is bigger than they if they decided to produce in the foreign country where the market is located. In a safe environment firms will generally opt to produce where costs are kept at a minimum, but since smaller than fair value compensation can be seen as a cost for the private firm, the "real" costs for producing in the foreign country might just be too big.

Appendix A

Appendix

Proof of Proposition 1. The value of the cash flows accruing to the safe monopolist, at time t, after investing are given by equation (3.6), and since the the expected value of the investment costs at the moment of investment are the investment costs themselves, we get that the expected value of the firm at the moment of investment is given by:

$$V_m^s\left(X, Q_m^s\right) = E\left[\int\limits_{t=0}^{\infty} \pi_m^s\left(t\right)e^{-rt}dt - \delta Q_m^s\right] = \Pi_m^s\left(X, Q_m^s\right) - \delta Q_m^s \tag{A.1}$$

Maximizing with respect to Q_m^s yields the optimal capacity choice for every given level of X,

$$Q_m^{s*}\left(X\right) = \frac{1}{2\eta}\left(1 - \frac{\left(r-\mu\right)\left(c_v - c_v\tau + r\delta\right)}{rX\left(\rho-1\right)\left(\tau-1\right)}\right) \tag{A.2}$$

Following standard real options technics [1] we get the value of the option to invest, noted by V_{0m}^s, by solving the the following ordinary differential equation (ode):

$$\frac{1}{2}\sigma^2 X^2 \frac{d^2 V_{0m}^s\left(X\right)}{dX^2} + \mu X \frac{dV_{0m}^s\left(X\right)}{dX} - rV_{0m}^s\left(X\right) = 0 \tag{A.3}$$

The solution for this ode is given by:

$$A_1 X^{\beta_1} + A_1 X^{\beta_2} \tag{A.4}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{A.5}$$

is the positive root and

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{A.6}$$

[1] Dixit and Pindyck (1994)

is the negative root of the quadratic:

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0 \tag{A.7}$$

Since we are talking about an investment option, we know that a natural boundary condition exists when $X \to 0$. When $X \to 0$ the value of the cash-flows go to zero so we have to make the value of the option be in line with this fact. And so, the term $A_2X^{\beta_2}$ must be dropped off since as $X \to 0$, because $\beta_2 < 0$, $A_2X^{\beta_2} \to \infty$, and the value of the option would go to infinity at the same time the value of the cash flow disappear, not making much economical sense. And so we have that the solution for the investment option is given by:

$$V_{0m}^s(X) = A_1X^{\beta_1} \tag{A.8}$$

Substitution of (A.2) into (A.1) and solving, with respect to X_m^{s*}, the smooth pasting (A.10) and value matching (A.9) conditions:

$$V_{0m}^s\left(X_m^{s*}\right) = V_m^s\left(X_m^{s*}, Q_m^{s*}(X_m^{s*})\right) \tag{A.9}$$

$$\left.\frac{\delta V_{0m}^s(X)}{\delta X}\right|_{X=X_m^{s*}} = \left.\frac{\delta V_m^s\left(X, Q_m^{s*}(X)\right)}{\delta X}\right|_{X=X_m^{s*}} \tag{A.10}$$

yields the value of the option to invest (3.9) and optimal time (trigger), given the optimal capacity, for investment for the safe monopolist:

$$X_m^{s*} = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)(c_v(1-\tau)+r\delta)}{r(1-\rho)(1-\tau)} \tag{A.11}$$

Substitution of (A.11) into (A.2) gives (3.11)

□

Proof of Proposition 2. The value of the cash flows accruing to the passive government, at time t, in a monopolistic context, after the monopolist invests, are given by equation (3.14), and since the government doesn't have investment costs, only collects taxes, its expected value at the moment the safe monopolist invests, noted by $G_m^s(X, Q_m^s)$ is the same as the expected value of its cash-flows:

$$G_m^s\left(X, Q_m^s\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{gm}^s(t)e^{-rt}dt\right] = \Pi_{gm}^s\left(X, Q_m^s\right) \tag{A.12}$$

Substitution of (3.11), the safe monopolist optimal quantity, into (A.12) gives (A.13), the value of the government taking into consideration the quantity in the market:

$$G_m^s(X) \equiv G_m^s\left(X, Q_m^{s*}\right) \tag{A.13}$$

For the value of the governments claim on the option of the monopolist, there's only need for one value matching condition (A.14) that states that the government claim on the investment option of the safe monopolist is equal to its claim on the operating business at the investment trigger X_m^{s*}:

$$G_{0m}^{\ s}\left(X_m^{s*}\right) = G_m^s\left(X_m^{s*}\right) \tag{A.14}$$

From (A.14) we get (3.17) □

Proof of Proposition 3. The value of the cash flows accruing to the safe follower, at time t, after investing are given by equation (3.18) and since the expected value of the investment costs at the moment of investment are simply the investment costs themselves, we get that the expected value of the safe follower at the moment of investment is given by:

$$V_f^s\left(X, Q_l^s, Q_f^s\right) = E\left[\int_{t=0}^{\infty} \pi_f^s\left(t\right) e^{-rt} dt - \delta Q_f^s\right] = \Pi_f^s\left(X, Q_l^s, Q_f^s\right) - \delta Q_f^s \tag{A.15}$$

Maximizing with respect to Q_f^s yields the optimal capacity choice for every given level of X,

$$Q_f^{s*}\left(X, Q_l^s\right) = \frac{1}{2\eta}\left(1 - \eta Q_l^s - \frac{(r-\mu)(c_v + r\delta - c_v\tau)}{rX(\rho-1)(\tau-1)}\right) \tag{A.16}$$

By substituting (A.16) into (A.15) we have:

$$V_f^s\left(X, Q_l^s\right) \equiv V_f^s\left(X, Q_l^s, Q_f^{s*}(X, Q_l^s)\right) \tag{A.17}$$

Solving, the smooth pasting (A.18) and value matching (A.19) conditions:

$$V_{0f}^s\left(X_f^{s*}, Q_l^s\right) = V_f^s\left(X_f^{s*}, Q_l^s\right) \tag{A.18}$$

$$\left.\frac{\delta V_{0f}^s\left(X, Q_l^s\right)}{\delta X}\right|_{X=X_f^{s*}} = \left.\frac{\delta V_f^s\left(X, Q_l^s\right)}{\delta X}\right|_{X=X_f^{s*}} \tag{A.19}$$

yields the value of the option to invest (3.21) and the optimal time (trigger), given the optimal capacity, for investment for the safe follower:

$$X_f^{s*}\left(Q_l^s\right) = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)\left(c_v\left(1-\tau\right)+r\delta\right)}{r\left(1-\rho\right)\left(1-\tau\right)\left(1-\eta Q_l^s\right)} \tag{A.20}$$

Substitution of (A.20) into (A.16) gives (3.23).

□

Proof of Proposition 4. The cash flows accruing to the safe leader, at time t, after investing and while monopolist, are given by equation (3.25), but because the follower will eventually enter the market, the the cash-flows in duopoly will need to take into consideration the extra quantity in the market, given by equation (3.26). Since that is the case, the expected value of the leader at the moment of investment needs to take into consideration that the follower will enter the market and that the value of the cash-flows will reduce. We get that the expected value of the safe leader at the moment of investment is given by:

$$
\begin{aligned}
V_{l_m}^{s}\left(X, Q_l^s\right) &= \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{l_m}^{i}(t)\, e^{-rt} dt - \delta Q_l^s\right] + \mathrm{E}\left[\int_{T_f^{i*}}^{\infty} \left(\pi_{l_d}^{i}(t) - \pi_{l_m}^{i}(t)\right) e^{-rt} dt\right] \\
&= \left(\frac{Q_l^s X(1-\eta Q_l^s)}{r-\mu}(1-\rho) - \frac{c_v Q_l^s}{r}\right)(1-\tau) \\
&\quad - \frac{X_f^{s*}(Q_l^s)\eta Q_f^s(Q_l^s) Q_l^s (\rho-1)(\tau-1)}{r-\mu}\mathrm{E}\left[e^{-rT_f^{e*}}\right] - \delta Q_l^s
\end{aligned}
\tag{A.21}
$$

Where [2]:

$$
E\left[e^{-rT_f^{s*}}\right] = \left(\frac{X}{X_f^{s*}(Q_l^s)}\right)^{\beta_1}
\tag{A.22}
$$

and T_f^{s*} is the expected first time when the stochastic process of demand reaches $X_f^{s*}(Q_l^s)$, the trigger of the follower. Substituting (A.22) into (A.21), and rearranging we get:

$$
V_{l_m}^{s}\left(X, Q_l^s\right) = \Pi_{l_m}^{s}\left(X, Q_l^s\right) - \delta Q_l^s
\tag{A.23}
$$

To find the optimal time to invest, X_p^{s*}, and the optimal capacity at the moment of investment, Q_l^{s*}, equations (A.24) and (A.25) need to be simultaneous solved numerically.

$$
V_{l_m}^{s}\left(X_p^{s*}, Q_l^{s*}\right) = V_{0f}^{s}\left(X_p^{s*}, Q_l^{s*}\right)
\tag{A.24}
$$

$$
\left.\frac{\partial V_{l_m}^{s}\left(X, Q_l^{s*}\right)}{\partial Q_l^{s*}}\right|_{X=X_p^{s*}} = 0
\tag{A.25}
$$

Equation (A.24) states that the leader's optimal time to invest, X_p^{s*}, considering that firms are symmetric and that there exists monopolistic rents, is when its value equals the option value of the follower. Equation (A.25) simply states that at that moment the leader will optimally choose its installed capacity Q_l^{s*}.

□

[2] Following Dixit and Pindyck (1994)

Proof of Proposition 5. The the cash-flows accruing to the passive government from the follower, at time t, are given by equation (3.41), and since the government doesn't have investment costs, only collects taxes, its expected value at the moment the safe follower invests, noted by $G_f^s(X, Q_l^s, Q_f^s)$, is the same as the expected value of its cash-flows:

$$G_f^s\left(X, Q_l^s, Q_f^s\right) = \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{g_f}^s(t) e^{-rt} dt\right] = \Pi_{g_f}^s\left(X, Q_l^s, Q_f^s\right) \tag{A.26}$$

Substitution of $Q_f^{s*}\left(Q_l^s\right)$ (3.23), the safe follower optimal quantity, and Q_l^{s*} the safe leader optimal quantity obtained in (3.32), into (A.26) gives (A.27), the value of the government taking into consideration the quantity in the market:

$$G_f^s\left(X\right) \equiv G_f^s\left(X, Q_l^{s*}, Q_f^{s*}(Q_l^{s*})\right) \tag{A.27}$$

For the value of the governments claim on the option of the follower, there's only need for one value matching condition (A.28) that states that the government claim on the investment option of the safe follower is equal to its claim on the operating business at the investment trigger X_f^{s*}:

$$G_{0}{}_f^s(X_f^{s*}) = G_f^s(X_f^{s*}) \tag{A.28}$$

From (A.28) we get (3.44).

The cash flows accruing to the passive government from the taxes collected from the leader, at time t, after the leader invests, and before the follower invests, are given by equation (3.34), but because the follower will eventually enter the market, the cash-flows from the taxes collected from the leader in duopoly will need to take into consideration the extra quantity in the market, given by equation (3.35). Since that is the case, the expected value of the government, from the taxes from the leader, at the moment the leader invests need to take into consideration that the follower will enter the market and that the value of the cash-flows from the leader will reduce. We get that the expected value of the government at the moment the leader invests is given by:

$$\begin{aligned}
G_{l_m}^s\left(X, X_f^s, Q_l^s, Q_f^s\right) &= \mathrm{E}\left[\int_{t=0}^{\infty} \pi_{g_{l_m}}^i(t)\, e^{-rt} dt\right] + \mathrm{E}\left[\int_{T_f^{i*}}^{\infty} \left(\pi_{g_{l_d}}^i(t) - \pi_{g_{l_m}}^i(t)\right) e^{-rt} dt\right] \\
&= \left(\frac{Q_l^s X(1-\eta Q_l^s)}{r-\mu} - \mathrm{E}\left[e^{-rT_f^{s*}}\right] \frac{X_f^s \eta Q_f^s Q_l^s}{r-\mu}\right)\rho \\
&\quad + \left(\left(\frac{Q_l^s X(1-\eta Q_l^s)}{r-\mu} - \mathrm{E}\left[e^{-rT_f^{s*}}\right] \frac{X_f^s \eta Q_f^s Q_l^s}{r-\mu}\right)(1-\rho) - \frac{c_v Q_l^s}{r}\right)\tau
\end{aligned} \tag{A.29}$$

Substituting (A.22), (A.20), (3.23) and the result from (3.32) into (A.29), and rearranging we get:

$$G^s_{l_m}(X) = \Pi^s_{g_{l_m}}\left(X, Q^{s*}_l\right) \tag{A.30}$$

For the value of the governments claim on the option of the leader, there's only need for one value matching condition (A.31) that states that the government claim on the investment option of the safe leader is equal to its claim on the operating business at the investment trigger X^{s*}_l:

$$G0^s_l(X^{s*}_l) = G^s_{l_m}(X^{s*}_l) \tag{A.31}$$

From (A.31) we get (3.40). □

Proof of Proposition 6. The cash flows accruing to the government, at time t, from operating the monopolist's business are given by equation (4.1). When the government expropriates the monopolist's business it pays a compensation relative to the value of the cash-flows that the monopolist would otherwise collect. We get that the expected value of the government at the moment it expropriates the monopolist is given by:

$$\begin{aligned} G_{e_m}(X, Q^e_m) &= \mathrm{E}\left[\int_{t=0}^{\infty}\left(\pi_{e_m}(t) - k\pi^e_m(t)\right)e^{-rt}dt\right] \\ &= \Pi_{e_m}\left(X, Q^e_m\right) - k\Pi^e_m\left(X, Q^e_m\right) \end{aligned} \tag{A.32}$$

We get the value of the option to expropriate by solving the the following ordinary differential equation (ode):

$$\frac{1}{2}\sigma^2X^2\frac{d^2G^e_m(X, Q^e_m)}{dX^2} + \mu X\frac{dG^e_m(X, Q^e_m)}{dX} - rG^e_m(X, Q^e_m) + \pi^e_{g_m} = 0 \tag{A.33}$$

The solution for this ode, and thus the value of the option to expropriate, considering it can be treated as an option to invest, is given by:

$$G^e_m(X, Q^e_m) = AX^{\beta_1} + \Pi^e_{g_m}\left(X, Q^e_m\right) \tag{A.34}$$

By solving the smooth pasting (A.36) and and value matching condition (A.35) for $X^*_{e_m}$:

$$G^e_m(X^*_{e_m}, Q^e_m) = G_{e_m}(X^*_{e_m}, Q^e_m) \tag{A.35}$$

$$\left.\frac{\delta G^e_m(X, Q^e_m)}{\delta X}\right|_{X=X^*_{e_m}} = \left.\frac{\delta G_{e_m}(X, Q^e_m)}{\delta X}\right|_{X=X^*_{e_m}} \tag{A.36}$$

we get the value of the government before expropriation (4.4) and the government's trigger to expropriate the monopolist:

$$X_{e_m}^{*}\left(Q_m^e\right)=\frac{\beta_1}{\left(\beta_1-1\right)}\frac{\left(r-\mu\right)\left(c_v k+c_v\left(1-k\right)\tau-c_g\right)}{r\left(1-\rho\right)\left(1-\tau\right)\left(1-k\right)\left(Q_m^e\eta-1\right)} \tag{A.37}$$

□

Proof of Proposition 7. The cash flows accruing to the risky monopolist, at time t, after investing are given by equation (3.6), but because the monopolist knows that eventually he will be expropriated, this cash-flows will eventually expire at the moment of expropriation. At expropriation the monopolist receives for compensation a fraction k of the present value of the cash-flows, as if it were to continue operation at that moment. We get that the expected value of the firm at the moment of investment is given by:

$$V_m^e\left(X,Q_m^e\right)=\mathrm{E}\left[\int_{t=0}^{T_{e_m}^*}\pi_m^e\left(t\right)e^{-rt}dt-\delta Q_m^e\right]+k\Pi_m^e\left(X_{e_m}^*\left(Q_m^e\right),Q_m^e\right)\mathrm{E}\left[e^{-rT_{e_m}^*}\right]$$

$$=\Pi_m^e\left(X,Q_m^e\right)\left(1-\left(\frac{X}{X_{e_m}^*(Q_m^e)}\right)^{\beta_1-1}\right)+k\Pi_m^e\left(X_{e_m}^*(Q_m^e),Q_m^e\right)\left(\frac{X}{X_{e_m}^*(Q_m^e)}\right)^{\beta_1}-\delta Q_m^e \tag{A.38}$$

where, following Dixit and Pindyck (1994), $\left(1-\left(\frac{X}{X_{e_m}^*(Q_l^e)}\right)^{\beta_1-1}\right)$ stands for the stochastic discount factor that corrects the present value of the cash-flows with expiration $T_{e_m}^*$, the expected time when the stochastic process of demand reaches $X_{e_m}^*$, the trigger for expropriation.

The solution for the value of the risky monopolist's investment option, similar to what was seen before, is given by:

$$V_{0m}^e\left(X,Q_l^e\right)=AX^{\beta_1} \tag{A.39}$$

Solving the value matching condition (A.40):

$$V_{0m}^e\left(X_m^{e*},Q_l^{e*}\right)=V_m^e\left(X_m^{e*},Q_l^{e*}\right) \tag{A.40}$$

gives us the value of the option to invest (4.9), and by numerically solving the smooth pasting (A.41) and the capacity optimization (A.42) conditions simultaneously, for $X_{e_m}^*$:

$$\left.\frac{\delta V_{0m}^e\left(X,Q_l^{e*}\right)}{\delta X}\right|_{X=X_m^{e*}}=\left.\frac{\delta V_m^e\left(X,Q_l^{e*}\right)}{\delta X}\right|_{X=X_m^{e*}} \tag{A.41}$$

$$\left.\frac{\delta V_m^e\left(X,Q_l^{e*}\right)}{\delta Q_l^{e*}}\right|_{X=X_m^{e*}}=0 \tag{A.42}$$

we get the trigger of the risky monopolist, X_m^{e*} and the optimal capacity at the moment of investment, Q_m^{e*}

□

Proof of Proposition 8. The cash flows accruing to the government, at time t, from operating the follower's business are given by equation (4.13). When the government expropriates the follower's business it pays a compensation relative to the value of the cash-flows that the follower would otherwise collect. We get that the expected value of the government at the moment it expropriates the monopolist is given by:

$$\begin{aligned} G_{e_f}(X, Q_l^e, Q_f^e) &= \mathrm{E}\left[\int\limits_{t=0}^{\infty} \left(\pi_{e_f}(t) - k\pi_f^e(t)\right) e^{-rt} dt\right] \\ &= \Pi_{e_f}\left(X, Q_l^e, Q_f^e\right) - k\Pi_f^e\left(X, Q_l^e, Q_f^e\right) \end{aligned} \tag{A.43}$$

Similar to the case of the monopolist, the value of the option to expropriate the follower is given by:

$$G_f^e(X, Q_l^e, Q_f^e) = AX^{\beta_1} + \Pi_{g_f}^e\left(X, Q_l^e, Q_f^e\right) \tag{A.44}$$

By solving the smooth pasting (A.45) and and value matching condition (A.46) for $X_{e_f}^*$:

$$G_f^e(X_{e_f}^*, Q_l^e, Q_f^e) = G_{e_f}(X_{e_f}^*, Q_l^e, Q_f^e) \tag{A.45}$$

$$\left.\frac{\delta G_f^e(X, Q_l^e, Q_f^e)}{\delta X}\right|_{X=X_{e_f}^*} = \left.\frac{\delta G_{e_f}(X, Q_l^e, Q_f^e)}{\delta X}\right|_{X=X_{e_f}^*} \tag{A.46}$$

we get the value of the government before expropriation (4.16) and the government's trigger to expropriate the follower:

$$X_{e_f}^*\left(Q_l^e, Q_f^e\right) = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\left(c_v k + c_v(1-k)\tau - c_g\right)}{r(1-\rho)(1-\tau)(1-k)\left((Q_l^e + Q_f^e)\eta - 1\right)} \tag{A.47}$$

□

Proof of Proposition 9. The cash flows accruing to the risky follower, at time t, after investing are given by equation (3.18), but because the follower knows that eventually he will be expropriated, this cash-flows will eventually expire at the moment of expropriation. At expropriation the follower receives for compensation a fraction k of the present value of the cash-flows, as if it were to continue operation at that moment. We get that the expected value of the firm at the moment of investment is given by:

$$
\begin{aligned}
V_f^e\left(X, Q_l^e, Q_f^e\right) &= \mathrm{E}\left[\int_{t=0}^{T_{e_f}^*} \pi_f^e(t)\, e^{-rt} dt - \delta Q_f^e\right] + k\Pi_f^e\left(X_{e_f}^*(Q_l^e, Q_f^e), Q_l^e, Q_f^e\right) \mathrm{E}\left[e^{-rT_{e_f}^*}\right] \\
&= \Pi_f^e\left(X, Q_l^e, Q_f^e\right)\left(1 - \left(\frac{X}{X_{e_f}^*(Q_l^e, Q_f^e)}\right)^{\beta_1 - 1}\right) \\
&\quad + k\Pi_f^e\left(X_{e_f}^*(Q_l^e, Q_f^e), Q_l^e, Q_f^e\right)\left(\frac{X}{X_{e_f}^*(Q_l^e, Q_f^e)}\right)^{\beta_1} - \delta Q_f^e
\end{aligned} \tag{A.48}
$$

where the first stochastic discount factor corrects the present value of the cash-flows with expiration $T_{e_f}^*$, the expected time when the stochastic process of demand reaches $X_{e_f}^*$, the trigger for expropriation.

The solution for the value of the risky follower's investment option, is given by:

$$
V0_f^e\left(X, Q_l^e, Q_f^e\right) = AX^{\beta_1} \tag{A.49}
$$

Solving the value matching condition (A.50):

$$
V0_f^e\left(X_f^{e*}, Q_l^e, Q_f^{e*}\right) = V_f^e\left(X_f^{e*}, Q_l^e, Q_f^{e*}\right) \tag{A.50}
$$

gives us the value of the option to invest (4.21), and by numerically solving the smooth pasting (A.51) and the capacity optimization (A.52) conditions simultaneously, for X_f^{e*}:

$$
\left.\frac{\delta V0_f^e\left(X, Q_l^e, Q_f^{e*}\right)}{\delta X}\right|_{X = X_f^{e*}} = \left.\frac{\delta V_f^e\left(X, Q_l^e, Q_f^{e*}\right)}{\delta X}\right|_{X = X_f^{e*}} \tag{A.51}
$$

$$
\left.\frac{\delta V_f^e\left(X, Q_l^e, Q_f^{e*}\right)}{\delta Q_f^{e*}}\right|_{X = X_f^{e*}} = 0 \tag{A.52}
$$

we get the trigger of the risky follower, $X_f^{e*}(Q_l^e)$ and the optimal capacity at the moment of investment, $Q_f^{e*}(Q_l^e)$

□

Proof of Proposition 10. **For values of $X_{e_{l_m}}^* < X_f^{e*}$**

The cash flows accruing to the government, at time t, from operating the leader's business, while monopolist, are given by equation (4.25), but because the follower will eventually will enter the market, the the cash-flows in duopoly will need to take into consideration the extra quantity in the market, given by equation (4.26). When the government expropriates the leader's business it pays a compensation relative to the value of the cash-flows that the leader would otherwise collect, accounting for the loss of value that would occur once the follower invested. We get that the expected value of the government at the moment it expropriates the leader is given by:

$$\begin{aligned} G_{e_{l_m}}(X, Q_l^e) &= \mathrm{E}\left[\int\limits_{t=0}^{\infty} \pi_{e_{l_m}}(t)\, e^{-rt} dt\right] + \mathrm{E}\left[\int\limits_{T_f^{e*}}^{\infty} \pi_{e_{l_d}}(t)\, e^{-rt} dt\right] - k\Pi_{l_m}^e\left(X, Q_l^e\right) \\ &= \Pi_{e_{l_m}}\left(X, Q_l^e\right) - k\Pi_{l_m}^e\left(X, Q_l^e\right) \end{aligned} \tag{A.53}$$

The value of the option to expropriate the leader, while monopolist, is given by:

$$G_{l_m}^e(X, Q_l^e) = AX^{\beta_1} + \Pi_{g_{l_m}}^e\left(X, Q_l^e\right) \tag{A.54}$$

By numerically solving the smooth pasting (A.56) and the value matching condition (A.55) for $X_{e_{l_m}}^*$:

$$G_{l_m}^e(X_{e_{l_m}}^*, Q_l^e) = G_{e_{l_m}}(X_{e_{l_m}}^*, Q_l^e) \tag{A.55}$$

$$\left.\frac{\delta G_{l_m}^e(X, Q_l^e)}{\delta X}\right|_{X=X_{e_{l_m}}^*} = \left.\frac{\delta G_{e_{l_m}}(X, Q_l^e)}{\delta X}\right|_{X=X_{e_{l_m}}^*} \tag{A.56}$$

we get the value of the government before expropriation (4.34) and the government's trigger to expropriate the leader while monopolist, $X_{e_{l_m}}^*(Q_l^e)$, taking in to consideration that we still need to find the leaders optimal quantity.

For values of $X_{e_{l_m}}^* > X_f^{e*}$

The cash flows accruing to the government, at time t, from operating the leader's business, while in a duopoly, are given by equation (4.26). When the government expropriates the leader's business it pays a compensation relative to the value of the cash-flows that the leader would otherwise collect in a duopoly. We get that the expected value of the government at the moment it expropriates the leader is given by:

$$G_{e_{l_d}}(X, Q_l^e) = \mathrm{E}\left[\int\limits_{t=0}^{\infty} \pi_{e_{l_d}}(t)\, e^{-rt} dt\right] - k\Pi_{l_d}^e\left(X, Q_l^e\right) = \Pi_{e_{l_d}}\left(X, Q_l^e\right) - k\Pi_{l_d}^e\left(X, Q_l^e\right) \tag{A.57}$$

The value of the option to expropriate the leader in a duopoly is given by:

$$G_{l_d}^e(X, Q_l^e) = AX^{\beta_1} + \Pi_{g_{l_d}}^e\left(X, Q_l^e\right) \tag{A.58}$$

By solving the smooth pasting (A.60) and the value matching condition (A.59) for $X^*_{e_{l_d}}$:

$$G^e_{l_d}(X^*_{e_{l_d}}, Q^e_l) = G_{e_{l_d}}(X^*_{e_{l_d}}, Q^e_l) \tag{A.59}$$

$$\left.\frac{\delta G^e_{l_d}(X, Q^e_l)}{\delta X}\right|_{X=X^*_{e_{l_d}}} = \left.\frac{\delta G_{e_{l_d}}(X, Q^e_l)}{\delta X}\right|_{X=X^*_{e_{l_d}}} \tag{A.60}$$

we get the value of the government before expropriation (4.34) and the government's trigger to expropriate the leader while in a duopoly:

$$X^*_{e_{l_d}}\left(Q^e_l\right) = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu)\left(c_v k + c_v (1-k)\tau - c_g\right)}{r(1-\rho)(1-\tau)(1-k)\left((Q^e_l + Q^{e*}_f(Q^e_l))\eta - 1\right)} \tag{A.61}$$

□

Proof of Proposition 11. **For values of $X^*_{e_{l_m}} < X^{e*}_f$**

The cash flows accruing to the risky leader while in monopolist, at time t, after investing are given by equation (3.25), and since the leader will eventually be expropriated, even before the follower invests, this cash-flows will expire at the moment of expropriation. At expropriation the leader receives for compensation a percentage k of the present value of the cash-flows, as if it were to continue operations at that moment, which means that the compensation will take into consideration the corrections for the eventual entrance of the follower in the market. We get that the expected value of the firm at the moment of investment is given by:

$$\begin{aligned} V^e_{l_m}\left(X, Q^e_l\right) &= \mathrm{E}\left[\int_{t=0}^{T^*_{e_{l_m}}} \pi^e_{l_m}(t)\, e^{-rt} dt - \delta Q^e_l\right] + k\Pi^e_{l_m}\left(X, Q^e_l\right)\mathrm{E}\left[e^{-rT^*_{e_{l_m}}}\right] \\ &= \Pi^e_m\left(X, Q^e_l\right)\left(1 - \left(\frac{X}{X^*_{e_{l_m}}(Q^e_l)}\right)^{\beta_1 - 1}\right) + k\Pi^e_{l_m}\left(X, Q^e_l\right)\left(\frac{X}{X^*_{e_{l_m}}(Q^e_l)}\right)^{\beta_1} - \delta Q^e_l \end{aligned} \tag{A.62}$$

where the first stochastic discount factor corrects the present value of the cash-flows with expiration $T^*_{e_{l_m}}$, the expected time when the stochastic process of demand reaches $X^*_{e_{l_m}}$, the trigger for expropriation.

For values of $X^*_{e_{l_m}} > X^{e*}_f$

The cash flows accruing to the risky leader while in duopoly, at time t, after investment are given by equation (3.26), and since the leader will be expropriated after the follower invests, we have this cash-flows will expire at the moment of expropriation. At expropriation the leader receives for compensation a percentage k of the present value of the cash-flows as if it were to continue operations in a duopoly. We get that the expected value of the firm at the moment of follower invest is given by:

$$V_{l_d}^e\left(X,Q_l^e\right)=\mathrm{E}\left[\int\limits_{t=0}^{T^*_{e_{l_d}}}\pi_{l_d}^e\left(t\right)e^{-rt}dt-\delta Q_l^e\right]+k\Pi_{l_d}^e\left(X,Q_l^e\right)\mathrm{E}\left[e^{-rT^*_{e_{l_d}}}\right]$$
$$=\Pi_{l_d}^e\left(X,Q_l^e\right)\left(1-\left(\frac{X}{X^*_{e_{l_d}}(Q_l^e)}\right)^{\beta_1-1}\right)+k\Pi_{l_d}^e\left(X^*_{e_{l_d}}(Q_l^e),Q_l^e\right)\left(\frac{X}{X^*_{e_{l_d}}(Q_l^e)}\right)^{\beta_1}-\delta Q_l^e \tag{A.63}$$

where the first stochastic discount factor corrects the present value of the cash-flows with expiration $T^*_{e_{l_d}}$, the expected time when the stochastic process of demand reaches $X^*_{e_{l_d}}$, the trigger for expropriation.

Before the follower invests, the value of the leader is the value of the cash-flows from being a monopolist in the market plus the the value of an option, that once exercised by the follower, will make him lose its monopolist rents and transform the market into a duopoly. The value of the leader before the follower invests is given by:

$$V_{l_m}^e\left(X,Q_l^e\right)=\Pi_m^e\left(X,Q_l^e\right)+AX^{\beta_1}-\delta Q_l^e \tag{A.64}$$

where by applying the necessary value matching condition:

$$V_{l_m}^e\left(X_f^{e*}\left(Q_l^e\right),Q_l^e\right)=V_{l_d}^e\left(X_f^{e*}\left(Q_l^e\right),Q_l^e\right) \tag{A.65}$$

we get:

$$A=\left(\Pi_{l_d}^e\left(X_f^{e*}\left(Q_l^e\right),Q_l^e\right)\left(1-\left(\frac{X_f^{e*}(Q_l^e)}{X^*_{e_{l_d}}(Q_l^e)}\right)^{\beta_1-1}\right)\right.$$
$$\left.+k\Pi_{l_d}^e\left(X^*_{e_{l_d}}(Q_l^e),Q_l^e\right)\left(\frac{X_f^{e*}(Q_l^e)}{X^*_{e_{l_d}}(Q_l^e)}\right)^{\beta_1}-\Pi_m^e\left(X_f^{e*}(Q_l^e),Q_l^e\right)\right)\left(\frac{1}{X_f^{e*}(Q_l^e)}\right)^{\beta_1} \tag{A.66}$$

Substitution of (A.66) into (A.64) gives (4.44).

For both cases:

Considering that firms are symmetric, and in both cases there is monopolistic rents, we get the leader's trigger for investment, X_p^{e*}, and its optimal capacity at the moment of investment, Q_l^{e*}, by numerically solving the value matching condition (A.67) and the capacity optimization condition (A.68) simultaneously, for X_p^{e*}:

$$V_{l_m}^e\left(X_p^{e*},Q_l^{e*}\right)=V_{0f}^e\left(X_p^{e*},Q_l^{e*}\right) \tag{A.67}$$

$$\left.\frac{\delta V_{l_m}^e\left(X,Q_l^e\right)}{\delta Q_l^{e*}}\right|_{X=X_p^{e*}}=0 \tag{A.68}$$

□

Proof of Proposition 12. The value of the cash flows accruing to the monopolist producing in its home country, at time t, after investing are given by equation (5.2), and since the the expected value of the investment costs at the moment of investment are the investment costs themselves, we get that the expected value of the firm at the moment of investment is given by:

$$V_m^h\left(X, Q_m^h\right) = E\left[\int\limits_{t=0}^{\infty} \pi_m^h(t)\, e^{-rt} dt - \delta Q_m^h\right] = \Pi_m^h\left(X, Q_m^h\right) - \delta Q_m^h \tag{A.69}$$

Maximizing with respect to Q_m^h yields the optimal capacity choice for every given level of X,

$$Q_m^{h*}(X) = \frac{1}{2\eta}\left(1 - \frac{(r-\mu)\left(c_h + r\delta_h + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right) P_h\right)}{rX(\rho-1)\left(\tau_f - 1\right)}\right) \tag{A.70}$$

The solution for the investment option is given by:

$$V_{0m}^h(X) = AX^{\beta_1} \tag{A.71}$$

Substitution of (A.70) into (A.69) and solving, with respect to X_m^{h*}, the smooth pasting (A.73) and value matching (A.72) conditions:

$$V_{0m}^s\left(X_m^{h*}\right) = V_m^s\left(X_m^{h*}, Q_m^{h*}(X_m^{h*})\right) \tag{A.72}$$

$$\left.\frac{\delta V_{0m}^h(X)}{\delta X}\right|_{X=X_m^{h*}} = \left.\frac{\delta V_m^h\left(X, Q_m^{h*}(X)\right)}{\delta X}\right|_{X=X_m^{h*}} \tag{A.73}$$

yields the value of the option to invest (5.5) and the optimal time (trigger), given the optimal capacity, for investment for the monopolist producing at home:

$$X_m^{h*} = \frac{\beta_1+1}{\beta_1-1}\frac{(r-\mu)\left(c_h + r\delta_h + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right) P_h\right)}{r(1-\rho)\left(1-\tau_f\right)} \tag{A.74}$$

Substitution of (A.74) into (A.70) gives (5.7)

□

Proof of Proposition 13. The value of the cash flows accruing to the follower producing in its home country, at time t, after investing are given by equation (5.9) and since the expected value of the investment costs at the moment of investment are simply the investment costs themselves, we get that the expected value of the safe follower at the moment of investment is given by:

$$V_f^h\left(X, Q_l^n, Q_f^h\right) = E\left[\int_{t=0}^{\infty} \pi_f^h(t)\, e^{-rt} dt - \delta Q_f^h\right] = \Pi_f^h\left(X, Q_l^n, Q_f^h\right) - \delta Q_f^h \tag{A.75}$$

Maximizing with respect to Q_f^h yields the optimal capacity choice for every given level of X,

$$Q_f^{h*}(X, Q_l^n) = \frac{1}{2\eta}\left(1 - \eta Q_l^n - \frac{(r-\mu)\left(c_h + r\delta_h + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right) P_h\right)}{rX(\rho-1)(\tau_f - 1)}\right) \tag{A.76}$$

By substituting (A.76) into (A.75) we have:

$$V_f^h(X, Q_l^n) \equiv V_f^h\left(X, Q_l^n, Q_f^{h*}(X, Q_l^n)\right) \tag{A.77}$$

Solving, the smooth pasting (A.79) and value matching (A.78) conditions:

$$V_{0f}^h\left(X_f^{h*}, Q_l^n\right) = V_f^h\left(X_f^{h*}, Q_l^n\right) \tag{A.78}$$

$$\left.\frac{\delta V_{0f}^h(X, Q_l^n)}{\delta X}\right|_{X = X_f^{h*}} = \left.\frac{\delta V_f^h(X, Q_l^n)}{\delta X}\right|_{X = X_f^{h*}} \tag{A.79}$$

yields the value of the option to invest (5.12) and the optimal time (trigger), given the optimal capacity, for investment for the follower producing at home:

$$X_f^{h*}(Q_l^n) = \frac{\beta_1 + 1}{\beta_1 - 1} \frac{(r-\mu)\left(c_h + r\delta + \gamma - (c_h+\gamma)\tau_h + \left(\theta(1-\tau_f) - \tau_f + \tau_h\right) P_h\right)}{r(1-\rho)(1-\tau_f)(1-Q_l^n)} \tag{A.80}$$

Substitution of (A.80) into (A.76) gives (5.14).

□

Proof of Proposition 14. The cash flows accruing to the leader producing at home, at time t, after investing and while monopolist, are given by equation (5.16), but because the follower will eventually will enter the market, the the cash-flows in duopoly will need to take into consideration the extra quantity in the market, given by equation (5.17). Since that is the case, the expected value of the leader at the moment of investment needs to take into consideration that the follower will enter the market and that the value of the cash-flows will reduce. We get that the expected value of the safe leader at the moment of investment is given by:

$$
\begin{aligned}
V^h_{l_m}\left(X, Q^h_l\right) &= \mathrm{E}\left[\int\limits_{t=0}^{\infty} \pi^h_{l_m}(t)\, e^{-rt}dt - \delta Q^h_l\right] + \mathrm{E}\left[\int\limits_{T^{n*}_f}^{\infty} \left(\pi^h_{l_d}(t) - \pi^h_{l_m}(t)\right) e^{-rt}dt\right] \\
&= Q^h_l \frac{P_h - c_h - \gamma}{r}(1-\tau_h) + Q^h_l \Bigg(\Bigg(\frac{X(1-\eta Q^h_l)}{r-\mu} \\
&\quad - \left(\frac{X}{X^{n*}_f(Q^h_l)}\right)^{\beta_1} \frac{X^{n*}_f(Q^h_l)\eta Q^{n*}_f(Q^h_l)}{r-\mu}\Bigg)(1-\rho) - \frac{P_h(1-\theta)}{r}\Bigg)\left(1-\tau_f\right) - \delta_h Q^h_l
\end{aligned}
\tag{A.81}
$$

where T^{n*}_f is the expected first time when the stochastic process of demand reaches $X^{n*}_f(Q^h_l)$, the trigger of the follower.

To find the optimal time to invest, X^{h*}_p, and the optimal capacity at the moment of investment, Q^{h*}_l, equations (A.82) and (A.83) need to be simultaneous solved numerically.

$$
V^h_{l_m}\left(X^{h*}_p, Q^{h*}_l\right) = V0^n_f\left(X^{h*}_p, Q^{h*}_l\right) \tag{A.82}
$$

$$
\left.\frac{\partial V^h_{l_m}\left(X, Q^{h*}_l\right)}{\partial Q^{h*}_l}\right|_{X = X^{h*}_p} = 0 \tag{A.83}
$$

□

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